Model Predictive Control for Optimal Pairs Trading Portfolio with Gross Exposure and Transaction Cost Constraints

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Abstract

Model Predictive Control (MPC) is a flexible yet tractable technique in control engineering that recently has gained much attention in the area of finance, particularly for its application to portfolio optimization. In this paper, we extend the MPC with linear feedback setting in Yamada and Primbs (2012) by incorporating the following two important and practical issues: The first issue is gross exposure (GE), which is the total value of long and short positions invested in risky assets (or stocks) as a proportion of the wealth possessed by a hedge fund. This quantity measures the leverage of a hedge fund, and the fund manager may limit the amount of leverage by imposing an upper bound, i.e., a GE constraint. The second issue is related to transaction costs, where the MPC algorithm may require frequent trades of many stocks leading to large transaction costs in practice. Here we assume that the transaction cost is proportional to the change in the amount of money (i.e., the change of absolute values of long or short positions) invested in each stock. We formulate the MPC strategy based on a conditional mean-variance problem which we show reduces to a convex quadratic problem, even with gross exposure and proportional transaction cost constraints. Based on numerical experiments using Japanese stock data, we demonstrate that the incorporation of the transaction cost constraint improves the empirical performance of the wealth in terms of Sharpe ratio, which may be improved further by adding the GE constraint.

Keywords: Pairs trading portfolio, Cointegration, Model predictive control, Conditional mean-variance optimization, Empirical simulations

1 Introduction

Pairs trading is a popular hedge fund investment strategy that constructs a long-short position between two stocks. Typically, the spread of a pair of stocks, i.e., one stock price minus some multiple of the other, is assumed to satisfy a predictive mean-reverting property characterized as cointegration (see, e.g., Engle and Granger (1987), Johansen (1991) for the notion of cointegration). An investor can make a profit by taking and clearing the long-short position replicating the spread. For example, such a spread position may be constructed by buying one stock and short selling the other, and is then cleared by the opposite trades if the spread reverts toward its mean level in a future period. Pairs trading is a useful strategy since it always leads to a positive profit as the spread converges, and its performance does not generally depend on the direction of the market.

A number of researchers have proposed quantitative models for pairs trading (see Elliott et al. (2005), Gatev et al. (2006), Do et al. (2006), Mudchanatongsuk et al. (2008), Tourin and Yan (2013), Song and Zhang (2013), Deshpande and Barmish (2016), Yamamoto and Hibiki (2017) and references therein). Also, we have developed an application of model predictive control (MPC) for portfolio optimization of cointegrated pairs of stocks in Yamada and Primbs (2012) and Primbs and Yamada (2017). MPC is a control methodology in which an open-loop finite horizon control problem that predicts some specified amount of time into the future is repeatedly solved on-line, but only the initial control action is implemented. Note that MPC is a flexible yet tractable method in control engineering, and recently it has gained attention for its application to portfolio optimization (see Piccoli and Marigo (2004), Herzog (2005), Herzog et al. (2006, 2007), Meindl (2006), Primbs and Sung (2008), Sridharan et al. (2011), Dombrovskii et al. (2004, 2005, 2006), Dombrovskii and Ob"edko (2011), Dombrovskii (2013) and Lee (2012), and references therein) as well as dynamic hedging in Meindl and Primbs (2004, 2008), Primbs (2010) and Bemporad et al. (2010, 2011, 2014).

The objective of this paper is to extend the MPC with linear feedback setting in Yamada and Primbs (2012) by incorporating the following two important issues: The first issue is related to transaction costs. The MPC algorithm may require frequent trades of many stocks leading to large transaction costs in practice. Here we assume that the transaction cost is proportional to the change in the amount of money invested in each stock (i.e., the change of absolute values of long or short positions). The second issue is gross exposure (GE), which is the total value of long and short positions invested in risky assets (or stocks) as a proportion of the wealth possessed by a hedge fund. This quantity measures the leverage of a hedge fund, and the fund manager may limit the amount of leverage by imposing an upper bound, i.e., a GE constraint (see Fan et al. (2012) and Qiu et al. (2015) for applications of GE constraints in portfolio optimization problems).

To this end, we first formulate a portfolio optimization problem involving cointegrated pairs of stocks following the result of Yamada and Primbs (2012), where the spread processes is expressed as a vector autoregressive (VAR) model and an MPC strategy is applied that calculates the conditional mean-variance (MV) optimal portfolio for a given prediction horizon at each step. Then, we extend the conditional MV problem to incorporate gross exposure and proportional transaction cost constraints. Although the solution in this case may not be derived in closed form, it is shown that the problem reduces to a convex quadratic optimization, even with the additional constraints described above, and can be solved efficiently to compute an optimal MPC input. Based on numerical experiments using Japanese stock data, we demonstrate that the incorporation of the transaction cost constraint improves the empirical performance of the wealth in terms of Sharpe ratio, and is even further improved by adding the GE constraint.

The rest of the paper is organized as follows: In Section 2, we explain the definition of cointegrated processes and formulate the MPC portfolio optimization problem for cointegrated pairs of stocks. In Section 3, we derive a solution to the conditional MV optimal portfolio for any given prediction horizon under which the spread process is expressed as a vector autoregressive (VAR) model. Then, we introduce a proportional transaction cost and a GE constraint in the conditional MV problem, and subsequently show that it can be solved as a convex quadratic optimization. In Section 5, we perform empirical simulations using stock price data consisting of the Nikkei 225 in Japan. The effects of the length of prediction horizon, rebalance intervals, transaction costs, and the GE constraint are all analyzed. Section 6 offers some concluding remarks.

2 Model predictive control for cointegrated pairs of stocks

2.1 Cointegrated pairs of stocks and the spread portfolio

To explain the definition of cointegrated pairs, we need to introduce the notion of integrated processes; A time series $\{X_t\}_{t=0,\ldots,N}$ is said to be integrated of order d, denoted by $X_t \sim I(d)$, if X_t is nonstationary but becomes stationary after differencing d times. In the context of stock markets, even though the price process X_t is a random walk (which is nonstationary), its integrated process may be stationary, i.e., $\Delta X_t := X_t - X_{t-1}$ is a stationary process, and therefore, $X_t \sim I(1)$ holds. Assume that there are two price processes satisfying $X_t \sim I(1)$ and $Y_t \sim I(1)$. If there exists a non-zero constant β such that $X_t - \beta Y_t$ is stationary, then X_t and Y_t are said to be cointegrated (see Engle and Granger (1987)). Assuming that X_t and Y_t stand for stock prices at time t, $X_t - \beta Y_t$ may be thought of as the spread of the two stocks which is realized by simultaneously investing in one unit of X_t

and $-\beta$ units of Y_t. If a pair of stocks is cointegrated, their spread is stationary and mean reverting, and we may take advantage of this property to construct a portfolio of multiple spreads of cointegrated pairs[1].

Suppose that there are m cointegrated pairs of stock prices,

$$\left(X_t^{(i)},\ Y_t^{(i)}\right),\ i = 1, \dots, m,$$

and consider constructing a portfolio consisting of the spreads, denoted by

$$S_t^{(i)} := X_t^{(i)} - \beta^{(i)} Y_t^{(i)}.$$

Let $u_t^{(i)},\ i \in \{1, \dots, m\}$ be a share unit invested in each spread, where the spread position consists of buying $X_t^{(i)}$ and short selling $\beta^{(i)} Y_t^{(i)}$ multiplied by the share unit $u_t^{(i)}$. Then, the wealth of the portfolio at time t, denoted by W_t, will evolve according to the following difference equation:

$$\begin{aligned} W_{t+1} &= \boldsymbol{u}_t^\top \boldsymbol{S}_{t+1} + (1+r)\left(W_t - \boldsymbol{u}_t^\top \boldsymbol{S}_t\right) \\ &= (1+r) W_t + \boldsymbol{u}_t^\top \left[\boldsymbol{S}_{t+1} - (1+r)\boldsymbol{S}_t\right] \end{aligned} \tag{2.1}$$

where r is the risk free interest rate applied for one period, and

$$\boldsymbol{S}_t := \left[S_t^{(1)}, \dots, S_t^{(m)}\right]^\top, \quad \boldsymbol{u}_t := \left[u_t^{(1)}, \dots, u_t^{(m)}\right]^\top \in \Re^m. \tag{2.2}$$

2.2 Model predictive control

In this paper, we consider the portfolio optimization problem over an infinite time horizon to (approximately) maximize the risk adjusted expected total return on wealth for pairs trading portfolio. To this end, we formulate a finite time control problem for any given prediction horizon, τ, and solve it repeatedly at each step in a receding-horizon fashion. Such a technique is known as model predictive receding-horizon control, or simply, "Model Predictive Control (MPC)" in the field of automatic control engineering[2].

Let $R_{t,\tau}$ be the total return on wealth over a given prediction horizon τ, i.e., $R_{t,\tau} := W_{t+\tau}/W_t$. Assume that the share vector $\boldsymbol{u}_t$ is adjusted at time t and stays constant until time $t+\tau$. Then the total return $R_{t,\tau}$ depends on $\boldsymbol{u}_t$ at time t only as

$$R_{t,\tau} = \frac{W_{t+\tau}}{W_t} = (1+r)^\tau + \frac{\boldsymbol{u}_t^\top}{W_t}\left[\boldsymbol{S}_{t+\tau} - (1+r)^\tau \boldsymbol{S}_t\right]. \tag{2.3}$$

Using the change of variable,

$$\boldsymbol{v}_t := \frac{\boldsymbol{u}_t}{W_t} \in \Re^m, \tag{2.4}$$

the conditional expectation of $R_{t,\tau}$ given the information up to time t, denoted $\mathbb{E}_t\left[R_{t,\tau}\right]$, may be written as

$$\mathbb{E}_t\left[R_{t,\tau}\right] = (1+r)^\tau + \boldsymbol{v}_t^\top \left[\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right) - (1+r)^\tau \boldsymbol{S}_t\right]. \tag{2.5}$$

Thus, we see that the conditional expectation of the total return may be controlled using the decision variable (or control input) $\boldsymbol{v}_t$.

Now, we formulate the following maximization problem over the decision variable vector $\boldsymbol{v}_t \in \Re^m$:

$$\max_{\boldsymbol{v}_t \in \Re^m} \left\{\mathbb{E}_t\left[R_{t,\tau}\right] - \frac{\gamma}{2} \cdot \mathbb{V}_t\left[R_{t,\tau}\right]\right\} \tag{2.6}$$

[1]See Yamada and Primbs (2012) for a selection procedure of cointegrated pairs from a given stock universe based on the Engle and Granger (1987) cointegration test.

[2]An alternative approach may be to formulate a dynamic optimization problem for a specified (possibly sufficiently long) terminal time period $T > 0$ as provided in Mudchanatongsuk et al. (2008), in which the spreads are modeled by the continuous time Ornstein-Uhlenbeck (OU) processes (Uhlenbeck and Ornstein (1930)) in log coordinates.

where $\mathbb{V}_t[R_{t,\tau}]$ denotes the conditional variance of $R_{t,\tau}$ such that

$$\mathbb{V}_t[R_{t,\tau}] = \mathbb{E}_t\left[(R_{t,\tau} - \mathbb{E}_t[R_{t,\tau}])^2\right]. \tag{2.7}$$

Here, the parameter γ is chosen to be positive and reflects the investor's risk aversion. Note that once the optimal control input $\boldsymbol{v}_t$ is obtained, the share vector may be recovered as $\boldsymbol{u}_t = W_t \boldsymbol{v}_t$.

The solution to the problem (2.6) provides a constant feedback law in the sense that $\boldsymbol{v}_t$ is constant in the time interval $[t,\ t+\tau)$ but has a feedback structure of the state variables in $\boldsymbol{S}_t$ up to time t. If we prefer not to rebalance frequently, we can use the same control input $\boldsymbol{v}_t$ (or equivalently, the same share vector $\boldsymbol{u}_t$) until time $t+\tau$. On the other hand, better performance may be expected by updating $\boldsymbol{u}_t$ according to changes in the state variables $\boldsymbol{S}_t$ and wealth level W_t as the current time t evolves. This is the basic idea of our MPC scheme for pairs trading, in which the resulting control law is dynamic in time and the current information with respect to these variables is incorporated at every rebalance period.

We are now in a position to describe the MPC algorithm as follows:

MPC algorithm

Step 0: For a specified terminal period $T > 0$, select $\delta > 0$ and $\tau > 0$ and subdivide the time interval $[0,\ T]$ as

$$0 < \delta < 2\delta < \cdots < (N-1)\delta < T \leq N\delta.$$

Let $n = 0$ and $t = 0$.

Step 1: Set $t_n = n\delta$.

1. If $t = t_n$, solve the problem (2.6) to find the optimal control input $\boldsymbol{u}_n^*$. Set $\boldsymbol{u}_t = \boldsymbol{u}_n^*$ and update $n \leftarrow n+1$.
2. If $t \neq t_n$, set $\boldsymbol{u}_t = \boldsymbol{u}_n^*$.

Step 2: Compute the wealth at time $t+1$ as

$$W_{t+1} = \boldsymbol{u}_t^\top \boldsymbol{S}_{t+1} + (1+r)\left(W_t - \boldsymbol{u}_t^\top \boldsymbol{S}_t\right). \tag{2.8}$$

Step 3: Update $t \leftarrow t+1$ and repeat from Step 1.

In the above algorithm, δ and $t_n = n\delta$ $(n = 0, 1, \ldots, N-1)$ determine the rebalance interval (which may also be referred to as the control horizon) and rebalance period, respectively, whereas τ provides the prediction horizon in the problem (2.6). These parameters may be chosen arbitrarily, but usually they satisfy $0 < \delta \leq \tau \leq T$ to guarantee optimality with respect to the prediction horizon at each time interval $[t_n,\ t_n+\delta]$. In particular, the trading strategy given by $\tau = 1$ may be referred to as a "Myopic strategy," in which only the single (and the shortest) period prediction is used.

3 Solution structure with VAR model

In this section, we derive the optimal control action in the MPC algorithm for pairs trading under the assumption that the spread process $\boldsymbol{S}_t$ follows the following VAR model[3],

$$VAR(q):\ \boldsymbol{S}_t = \Phi_1 \boldsymbol{S}_{t-1} + \cdots + \Phi_q \boldsymbol{S}_{t-q} + \boldsymbol{e}_t, \quad \Phi_i \in \Re^{m\times m} \tag{3.1}$$

[3]Note that the control technique demonstrated in this paper may be applied in the case where the spread process is given by a vector AR moving average (VARMA) model and can easily be extended to the case of vector cointegration in Johansen (1991), although we omit the details.

where $\boldsymbol{e}_t$ is an m-dimensional white noise process with a covariance matrix $\Sigma \in \Re^{m\times m}$, and $\Phi_i \in \Re^{m\times m}$, $i = 1,\dots,q$ a coefficient matrix. Without loss of generality, one can assume that $\mathbb{E}[\boldsymbol{e}_t] = 0$ by replacing $\boldsymbol{S}_{t-i}$ in (3.1) with $\boldsymbol{S}_{t-i} - \mu_0$ for $i = 0,1,\dots,q$, where $\mu_0 \in \Re^m$ is the unconditional expectation vector of $\boldsymbol{S}_t$.

3.1 Conditional mean and variance for $q = 1$

To illustrate the parameter specification, we first consider the case $q = 1$ in (3.1), where the spread process follows

$$VAR(1): \ \boldsymbol{S}_t = \Phi_1 \boldsymbol{S}_{t-1} + \boldsymbol{c} + \boldsymbol{e}_t, \tag{3.2}$$

with $\Phi_1 \in \Re^{m\times m}$ and $\boldsymbol{c} \in \Re^m$. By applying (3.2) recursively, $\boldsymbol{S}_{t+\tau}$ may be expressed as

$$\begin{aligned} \boldsymbol{S}_{t+\tau} &= \Phi_1 \boldsymbol{S}_{t+\tau-1} + \boldsymbol{c} + \boldsymbol{e}_{t+\tau} \\ &= \Phi_1^2 \boldsymbol{S}_{t+\tau-2} + (\Phi_1 + I)\,\boldsymbol{c} + \Phi_1 \boldsymbol{e}_{t+\tau-1} + \boldsymbol{e}_{t+\tau} \\ &= \quad \cdots \quad \cdots \quad \cdots \\ &= \Phi_1^\tau \boldsymbol{S}_t + \left(\Phi_1^{\tau-1} + \cdots + \Phi_1 + I\right)\boldsymbol{c} + \Phi_1^{\tau-1}\boldsymbol{e}_{t+1} + \cdots + \Phi_1 \boldsymbol{e}_{t+\tau-1} + \boldsymbol{e}_{t+\tau}. \end{aligned} \tag{3.3}$$

Then, the conditional expectation of $\boldsymbol{S}_{t+\tau}$ is specified as

$$\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right) = \Phi_1^\tau \boldsymbol{S}_t + \left(\Phi_1^{\tau-1} + \cdots + \Phi_1 + I\right)\boldsymbol{c}. \tag{3.4}$$

By substituting (3.4) into (2.5), we see that $\mathbb{E}_t[R_{t,\tau}]$ can be represented as a linear function of $\boldsymbol{S}_t$.

Next, we derive the conditional variance $\mathbb{V}_t[R_{t,\tau}]$. Since

$$\begin{aligned} R_{t,\tau} - \mathbb{E}_t[R_{t,\tau}] &= \boldsymbol{v}_t^\top \left[\boldsymbol{S}_{t+\tau} - \mathbb{E}_t(\boldsymbol{S}_{t+\tau})\right] \\ &= \boldsymbol{v}_t^\top \left(\Phi_1^{\tau-1}\boldsymbol{e}_{t+1} + \cdots + \Phi_1 \boldsymbol{e}_{t+\tau-1} + \boldsymbol{e}_{t+\tau}\right) \end{aligned}$$

holds from conditions (2.5), (3.3), and (3.4), we have

$$\begin{aligned} \mathbb{V}_t[R_{t,\tau}] &= \mathbb{E}_t\left[\left(R_{t,\tau} - \mathbb{E}_t[R_{t,\tau}]\right)^2\right] \\ &= \boldsymbol{v}^\top \left(\Sigma + \Psi_1 \Sigma \Psi_1^\top + \cdots + \Psi_{\tau-1}\Sigma\Psi_{\tau-1}^\top\right)\boldsymbol{v}, \end{aligned} \tag{3.5}$$

where Ψ_i is defined as

$$\Psi_i := \Phi_1^i, \ i = 1,\dots,\tau-1. \tag{3.6}$$

Consequently, we see that the conditional mean and variance of $R_{t,\tau}$ are represented using parameters of the $VAR(1)$ model in (3.2).

3.2 General case and the optimal control input

In the case of $q \geq 2$ in (3.1), we can obtain the conditional mean and variance by constructing an augmented system as

$$\widehat{\boldsymbol{S}}_t = \widehat{\Phi}_1 \widehat{\boldsymbol{S}}_{t-1} + \widehat{\boldsymbol{c}} + \widehat{\boldsymbol{e}}_t, \tag{3.7}$$

where

$$\widehat{\boldsymbol{S}}_t := \left[\boldsymbol{S}_t^\top, \boldsymbol{S}_{t-1}^\top, \dots, \boldsymbol{S}_{t-q+1}^\top\right]^\top \tag{3.8}$$

and

$$\widehat{\Phi}_1 = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{q-1} & \Phi_q \\ I_m & 0 & \cdots & 0 & 0 \\ 0 & I_m & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_m & 0 \end{bmatrix}, \quad \widehat{\boldsymbol{c}} := \begin{bmatrix} \boldsymbol{c} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \widehat{\boldsymbol{e}}_t := \begin{bmatrix} \boldsymbol{e}_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{3.9}$$

In this case, all the coefficient parameters may be derived similar to the above mentioned $VAR(1)$ model. For example, since

$$\begin{aligned}\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right) &= \left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\times\mathbb{E}_t\left(\widehat{\boldsymbol{S}}_{t+\tau}\right)\\ &= \left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\times\left\{\widehat{\Phi}_1^{\tau}\widehat{\boldsymbol{S}}_t+\left(\widehat{\Phi}_1^{\tau-1}+\cdots+\widehat{\Phi}_1+I\right)\widehat{\boldsymbol{c}}\right\}. \end{aligned} \tag{3.10}$$

holds from condition (3.4), we have that

$$\begin{aligned}R_{t,\tau}-\mathbb{E}_t\left[R_{t,\tau}\right] &= \boldsymbol{v}_t^{\top}\left[\boldsymbol{S}_{t+\tau}-\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right)\right]\\ &= \boldsymbol{v}_t^{\top}\left\{\left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\times\left[\widehat{\boldsymbol{S}}_{t+\tau}-\mathbb{E}_t\left(\widehat{\boldsymbol{S}}_{t+\tau}\right)\right]\right\}\\ &= \boldsymbol{v}_t^{\top}\left\{\left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\times\left(\widehat{\Phi}_1^{\tau-1}\widehat{\boldsymbol{e}}_{t+1}+\cdots+\widehat{\Phi}_1\widehat{\boldsymbol{e}}_{t+\tau-1}+\widehat{\boldsymbol{e}}_{t+\tau}\right)\right\}\end{aligned}$$

and hence the conditional variance $\mathbb{V}_t\left[R_{t,\tau}\right]$ may be obtained using the same representation as that in (3.5) with

$$\Psi_i=\left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\widehat{\Phi}_1^{i}\left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]^{T},\qquad i=1,\dots,\tau-1. \tag{3.11}$$

Consequently, the problem (2.6) may be rewritten as follows:

$$\max_{\boldsymbol{v}_t\in\Re^m}\left\{\boldsymbol{v}_t^{\top}\left[\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right)-(1+r)^{\tau}\boldsymbol{S}_t\right]-\frac{\gamma}{2}\cdot\boldsymbol{v}_t^{\top}\left(\Sigma+\Psi_1\Sigma\Psi_1^{\top}+\cdots+\Psi_{\tau-1}\Sigma\Psi_{\tau-1}^{\top}\right)\boldsymbol{v}_t\right\} \tag{3.12}$$

with Ψ_i defined in (3.11). By applying the first order condition, the optimal control input, $\boldsymbol{v}_t=\boldsymbol{v}_t^*$, is obtained to be

$$\begin{aligned}\boldsymbol{v}_t^* &= \frac{1}{\gamma}\left(\Sigma+\Psi_1\Sigma\Psi_1^{\top}+\cdots+\Psi_{\tau-1}\Sigma\Psi_{\tau-1}^{\top}\right)^{-1}\\ &\quad\times\left[\left[\begin{array}{cccc} I_m & 0 & \dots & 0\end{array}\right]\times\left\{\widehat{\Phi}_1^{\tau}\widehat{\boldsymbol{S}}_t+\left(\widehat{\Phi}_1^{\tau-1}+\cdots+\widehat{\Phi}_1+I\right)\widehat{\boldsymbol{c}}\right\}-\left(1+r_\tau\right)^{\tau}\boldsymbol{S}_t\right]\end{aligned} \tag{3.13}$$

where condition (3.10) has been substituted for $\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right)$ in (3.12).

Although the optimal control input $\boldsymbol{v}_t^*$ is supposed to be constant over the time horizon to $t+\tau$ in the problem (3.12), and provides a constant feedback law as stated in Subsection 2.2, it is worthwhile to mention that $\boldsymbol{v}_t^*$ becomes linear feedback based on the MPC algorithm. To see this, we first note that all the coefficient matrices of $\widehat{\boldsymbol{S}}_t=\left[\boldsymbol{S}_t^{\top},\boldsymbol{S}_{t-1}^{\top},\dots,\boldsymbol{S}_{t-q+1}^{\top}\right]^{\top}$ and $\boldsymbol{S}_t$ are constant in (3.13). Then, there exist constant matrices $F_i\in\Re^{m\times m},\ i=1,\dots,m$ and a vector $\boldsymbol{g}\in\Re^m$ so that $\boldsymbol{v}_t^*$ is expressed as

$$\boldsymbol{v}_t^*=F_1\boldsymbol{S}_t+F_2\boldsymbol{S}_{t-1}+\cdots+F_q\boldsymbol{S}_{t-q+1}+\boldsymbol{g}. \tag{3.14}$$

Condition (3.14) indicates that $\boldsymbol{v}_t^*$ is linearly dependent on $\boldsymbol{S}_t,\ \boldsymbol{S}_{t-1},\dots,\boldsymbol{S}_{t-q-1}$ and may be updated when a new state variable, say $\boldsymbol{S}_{t+1}$ is observed by applying the MPC algorithm. Consequently, we see that the usage of the solution of (3.12) combined with the MPC algorithm leads to a linear feedback controller in the state variables, $\boldsymbol{S}_t,\ \boldsymbol{S}_{t-1},\dots,\boldsymbol{S}_{t-q+1}$.

4 Transaction cost and gross exposure constraints

Here we discuss two important issues for the practical application of pairs trading using the MPC algorithm provided in this paper. The first issue relates to transaction costs. This is because the MPC algorithm of this paper may require frequent trading of many stocks, which is likely to result in a large amount of transaction costs in practice. The second issue concerns gross exposure, which corresponds to the total size of the positions exposed to risk in the stock market compared to the amount of wealth possessed by the fund. If the gross exposure is too large, such a portfolio may be significantly affected by market fluctuations and be difficult to control.

4.1 Consideration of transaction costs

In this paper, we consider a proportional transaction cost which is incurred every time a trade occurs in each stock. The main source of such a transaction cost is the bid-ask spread between the quoted prices of sell and buy orders. In exchange stock markets, the true price of a stock may be considered somewhere between the bid and ask prices, and one needs to pay an additional cost for selling or buying the stock in terms of the quoted prices in the market. Assuming that the true price is given by the mean value of bid and ask prices, one half of the bid-ask spread may be considered as a transaction cost for selling or buying the stock. Since the bid-ask spread is related to the minimum tick size and the minimum tick size is usually approximately proportional to the quoted prices, it is reasonable to consider that such a transaction cost is proportional to the transaction price times the transaction volume executed when rebalancing.

Let $\Delta u_t^{(i)} = u_t^{(i)} - u_{t-}^{(i)}$ denote the change in shares for the i-th spread $S_t^{(i)} = X_t^{(i)} - \beta^{(i)} Y_t^{(i)},\ i = 1, \ldots, m$, where $u_{t-}^{(i)}$ represents the shares possessed in the i-th spread immediately prior to the rebalance period $t = t_n = n\delta,\ n = 0, 1, \ldots, N-1$. Then, the proportional transaction cost related to trading the spread $S_t^{(i)}$ may be modeled as

$$\rho \Gamma_t^{(i)} \left| \Delta u_t^{(i)} \right|, \quad \Gamma_t^{(i)} := \left| X_t^{(i)} \right| + \left| \beta^{(i)} Y_t^{(i)} \right|, \quad i = 1, \ldots, m. \tag{4.1}$$

where $\rho > 0$ is a given transaction cost rate. For example, if the investment value in X_t is adjusted from $u_{t-}^{(i)} X_t$ to $u_t^{(i)} X_t$, the transaction cost is assumed to be $\rho \left| u_t^{(i)} X_t - u_{t-}^{(i)} X_t \right|$.

Under the assumption of a proportional transaction cost, the sum of $\rho \Gamma_t^{(i)} \left| \Delta u_t^{(i)} \right|$ is deducted from the wealth when rebalancing at time t, and the value of the wealth at time $t + \tau$ may be given as follows:

$$W_{t+\tau} = \boldsymbol{u}_t^\top \boldsymbol{S}_{t+\tau} + (1+r)^\tau \left(W_t - \boldsymbol{u}_t^\top \boldsymbol{S}_t - \sum_{i=1}^m \rho \Gamma_t^{(i)} \left| \Delta u_t^{(i)} \right| \right). \tag{4.2}$$

Using the change of variable $\boldsymbol{v}_t = \boldsymbol{u}_t / W_t$, the total return $R_{t,\tau} = W_{t+\tau}/W_t$ is obtained as

$$R_{t,\tau} = (1+r)^\tau + \boldsymbol{v}_t \left[\boldsymbol{S}_{t+\tau} - (1+r)^\tau \boldsymbol{S}_t \right] - (1+r)^\tau \sum_{i=1}^m \rho \Gamma_t^{(i)} \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right|, \tag{4.3}$$

where $v_t^{(i)}$ denotes the i-th entry of $\boldsymbol{v}_t$.

We show that the problem (2.6) with $R_{t,\tau}$ in (4.3) boils down to a quadratic programming problem. For this purpose, let us replace the absolute value terms in (4.3) by a set of new variables $\kappa_t^{(i)},\ i = 1, \ldots, m$ and rewrite $R_{t,\tau}$ in (4.3) as follows:

$$R_{t,\tau} = (1+r)^\tau + \boldsymbol{v}_t \left[\boldsymbol{S}_{t+\tau} - (1+r)^\tau \boldsymbol{S}_t \right] - (1+r)^\tau \sum_{i=1}^m \rho \Gamma_t^{(i)} \kappa_t^{(i)}, \tag{4.4}$$

where $\kappa_t^{(i)}$ satisfies

$$\kappa_t^{(i)} = \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right|. \tag{4.5}$$

In this case, we need to solve the maximization problem (2.6) subject to $R_{t,\tau}$ in (4.4) together with the equality constraint (4.5). But, in fact, condition (4.5) may equivalently be replaced by the following inequality constraint:

$$\kappa_t^{(i)} \geq \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right| \quad \Leftrightarrow \quad -\kappa_t^{(i)} \leq v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \leq \kappa_t^{(i)}, \tag{4.6}$$

Note that the transaction cost terms effect the conditional expectation of $R_{t,\tau}$, i.e.,

$$\mathbb{E}_t \left[R_{t,\tau} \right] = (1+r)^\tau + \boldsymbol{v}_t^\top \left[\mathbb{E}_t \left(\boldsymbol{S}_{t+\tau} \right) - (1+r)^\tau \boldsymbol{S}_t \right] - (1+r)^\tau \sum_{i=1}^m \rho \Gamma_t^{(i)} \kappa_t^{(i)}$$

but the conditional variance is unchanged from the one without the transaction cost constraint. Therefore, assuming that $\boldsymbol{S}_t$ follows a $VAR(q)$ model in (3.1), the conditional mean and variance optimization problem can be reformulated as follows:

$$\begin{aligned} \max \quad & \boldsymbol{v}_t^\top \left[\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right) - (1+r)^\tau \boldsymbol{S}_t\right] - (1+r)^\tau \sum_{i=1}^m \rho_c \Gamma_t^{(i)} \kappa_t^{(i)} - \frac{\gamma}{2} \cdot \boldsymbol{v}_t^\top \left(\Sigma + \Psi_1 \Sigma \Psi_1^\top + \cdots + \Psi_{\tau-1}\Sigma\Psi_{\tau-1}^\top\right) \boldsymbol{v}_t \\ \text{s.t.} \quad & v_t^{(i)}, \kappa_t^{(i)} \in \Re, \quad -\kappa_t^{(i)} \le v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \le \kappa_t^{(i)}, \ i = 1, \ldots, m. \end{aligned} \tag{4.7}$$

where $\mathbb{E}_t\left(\boldsymbol{S}_{t_\tau}\right)$ and $\Psi_i, \ i = 1, \ldots, \tau - 1$ are, respectively, given in (3.10) and (3.11). Clearly, the problem (4.7) is a convex quadratic programming problem in which the objective function is quadratic whereas the constraint is linear in $\boldsymbol{v}_t \in \Re^m$. In the problem (4.7), note that we use a different notation ρ_c instead of ρ since the term with ρ_c can be thought of as a turnover penalty related to the transaction cost. One may choose $\rho_c > \rho$ if the investor is concerned about a large transaction cost resulting from high turnover, or set $\rho_c = 0$ if it is not of concern. A reasonable choice is to select $\rho_c = \rho$ to associate the turnover penalty with the actual (proportional) transaction cost.

4.2 Gross exposure constraint

The gross exposure (GE) is the proportion of the total value of long and short positions invested in risky assets (or stocks) as a fraction of the value of the wealth possessed by the hedge fund. In fact, the GE may be used to measure the size of leverage, and so, the fund manager is able to limit the amount of leverage by imposing an upper bound on the GE condition.

In our context, since the value of long and short positions invested in the i-th spread is written as

$$\left|u_t^{(i)}\right| \left(\left|X_t^{(i)}\right| + \left|\beta^{(i)} Y_t^{(i)}\right|\right) = \left|u_t^{(i)}\right| \Gamma_t^{(i)},$$

using $\Gamma_t^{(i)}$ in (4.1), the GE at time t may be defined by the following quantity:

$$\sum_{i=1}^m \frac{\left|u_t^{(i)}\right| \Gamma_t^{(i)}}{W_t} = \sum_{i=1}^m \left|v_t^{(i)}\right| \Gamma_t^{(i)}. \tag{4.8}$$

Subsequently, the GE constraint is given as follows:

$$\sum_{i=1}^m \left|v_t^{(i)}\right| \Gamma_t^{(i)} \le \lambda, \tag{4.9}$$

where $\lambda > 0$ is an upper bound on the GE condition.

It can be shown that the GE constraint (4.9) is transferred to a linear constraint similar to the transaction cost condition in (4.4), i.e., the GE constraint (4.9) can be rewritten as

$$\sum_{i=1}^m \zeta_t^{(i)} \Gamma_t^{(i)} \le \lambda, \quad -\zeta_t^{(i)} \le v_t^{(i)} \le \zeta_t^{(i)} \tag{4.10}$$

by introducing a new variable $\zeta_t^{(i)}$. Then the conditional MV problem with the transaction cost and GE constraints is formulated as follows:

$$\begin{aligned} \max \quad & \boldsymbol{v}_t^\top \left[\mathbb{E}_t\left(\boldsymbol{S}_{t+\tau}\right) - (1+r)^\tau \boldsymbol{S}_t\right] - (1+r)^\tau \sum_{i=1}^m \rho_c \Gamma_t^{(i)} \kappa_t^{(i)} - \frac{\gamma}{2} \cdot \boldsymbol{v}_t^\top \left(\Sigma + \Psi_1 \Sigma \Psi_1^\top + \cdots + \Psi_{\tau-1}\Sigma\Psi_{\tau-1}^\top\right) \boldsymbol{v}_t \\ \text{s.t.} \quad & v_t^{(i)}, \kappa_t^{(i)}, \zeta_t^{(i)} \in \Re, \quad -\kappa_t^{(i)} \le v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \le \kappa_t^{(i)}, \\ & \sum_{i=1}^m \zeta_t^{(i)} \Gamma_t^{(i)} \le \lambda, \quad -\zeta_t^{(i)} \le v_t^{(i)} \le \zeta_t^{(i)}, \ i = 1, \ldots, m. \end{aligned} \tag{4.11}$$

We see that the conditional MV problem in (4.11) is still a convex quadratic programming, even with transaction cost and GE constraints.

5 Empirical simulation

In this section, we perform empirical simulations using stock price data consisting of the Nikkei 225 in Japan, and illustrate the performance of our proposed MPC algorithm for pairs trading in a variety of settings.

5.1 Data description and assumptions

Consider the Japanese stock market consisting of the Nikkei 225 as of the end of September 2016, in which 218 company names having at least 5 years of consecutive daily price data are assumed to be our investment universe to ensure the data availability. Since we intend to examine the effects of the length of prediction horizon, rebalance intervals and transaction costs given that the selected pairs are expected to be cointegrated in the simulation period as well, we split the data period used for pairs selection into a parameter estimation period and a simulation period. As a first step, we used the final 3 year period, October 2013 through September 2016, and selected 27 cointegrated pairs using the selection procedure in Yamada and Primbs (2012)[4]. Then, parameter values of the VAR model were estimated using the first 2 year data period, whereas the following 1 year data period was reserved for the out-of-sample simulation. Note that we have run the same simulations as described in this section for other 3 year periods, e.g., October 2012 through September 2015 based on our stock universe, with the results being quite similar. Thus, here we limit our focus to the simulation results for the data period of October 2013 through September 2016 only.

In addition to the estimation of the parameters in the VAR model and those in the spreads, there are several parameters to be chosen in the MPC algorithm, such as the risk aversion coefficient γ, the prediction horizon τ, the rebalance interval δ, and the GE constraint level λ. Among these parameters, the GE constraint λ should be set by the fund a priori to reflect their leverage limit. In the case of pairs trading, the amount of long positions in stocks tends to be equal to that for short positions so that the net position is zero. Thus, if a fund would not like to take short positions in risky assets that exceed 40% of their wealth (or similarly for long positions), the GE should be less than 80% or $\lambda = 0.8$. In this section, we set $\lambda = 1$ (which is a little greater than the example mentioned above) when the GE constraint is imposed in the MPC algorithm, and the risk free rate for one day and the initial wealth of the portfolio are taken to be $r = 0.01/245$ and $W_0 = 1$, respectively. In the following two subsections, we assume that the risk aversion coefficient is given as $\gamma = 1 \times 10^3$, whereas the relation between the choice of γ and the portfolio performance is explored in Subsection 5.4 in comparison with the standard (unconditional) MV optimal portfolio.

5.2 Effect of prediction horizon and transaction cost

First, we examine the effects of the length of the prediction horizon τ and the transaction cost parameter ρ_c on the wealth performance of MPC. Specifically, we apply the MPC algorithm for the 27 pairs chosen in the 3 years data period, where the required parameters are estimated in the first 2 years and the wealth performance is evaluated on the remaining 1 year. Also, we assume that rebalancing is performed every day, i.e., the rebalance interval is given by $\delta = 1$.

The relation between Sharpe ratio and prediction horizon τ is shown in Fig. 5.1, where the problem (4.11) is solved with the GE constraint parameter $\lambda = 1$ and the proportional transaction cost is deducted from the wealth at every rebalance period according to $\rho = 0$–0.5% (i.e., the transaction cost rate is varied from 0bp to 50bp).

[4] See Appendix A for the list of companies in the 27 pairs.

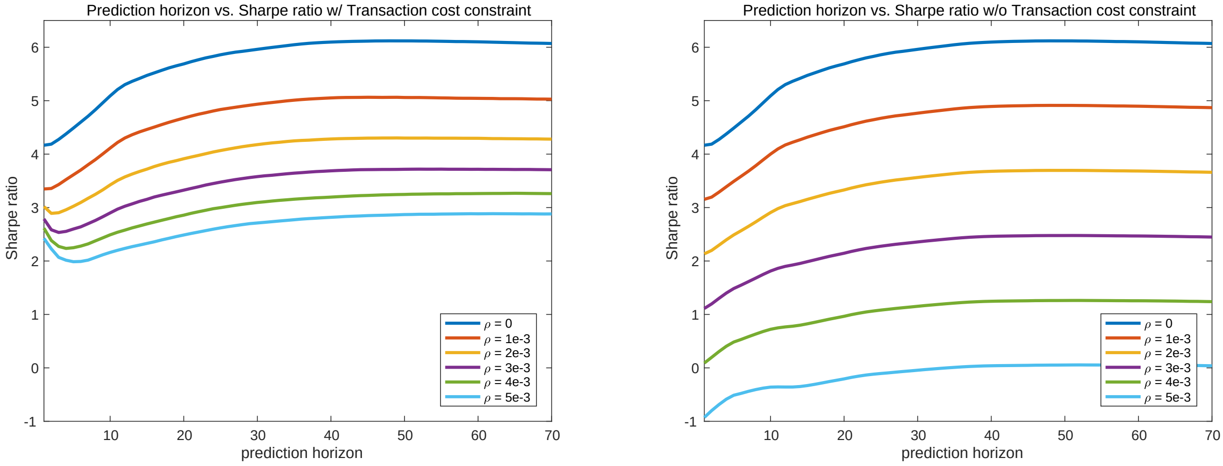

Fig. 5.1: Effect of the length of prediction horizon and the transaction cost on the wealth performance. The left hand side plots the Shape ratio for MPC with transaction cost constraint and the right hand side without transaction cost constraint.

The left hand side plot is obtained by solving the problem (4.11) with $\rho_c = \rho$, whereas the right hand side uses $\rho_c = 0$ so that the problem is solved as if there is no transaction cost, but it is deducted in the simulation. If we compare the graphs in both sides, we first observe that the Sharpe ratio decreases with a larger ρ. In particular, it becomes negative with $\rho = 0.5\%$ in the right hand side where $\rho_c = 0$. On the other hand, the decrease in Sharpe ratio is slower in the right hand side, and stays above or around 2 even with the largest transaction cost rate of $\rho\ (= \rho_c) = 0.5\%$.

Another feature that can be observed from Fig. 5.1 is that the Sharpe ratio tends to increase with the length of the prediction horizon, say up to $\tau = 40$–50. In particular, the increase in Sharpe ratio is higher if we set $\rho_c = 0$, although the Sharpe ratio drops a little when $\rho_c\ (= \rho) = 0.3$–0.5% and τ is small (say, τ is less than 5 or so). A future interesting topic is to clarify the theoretical mechanism in which the length of τ actually contributes to the improvement of the Sharpe ratio.

5.3 Effect of rebalance interval and GE constraint

Next, we investigate the effect of the rebalance interval and GE constraint. As shown in Fig. 5.2, the Sharpe ratio was computed as a function of the rebalance interval δ for a prediction horizon of $\tau = 40$ and various values of the prediction horizon, The left hand side denotes the results when $\rho_c = \rho = 0$–0.5% is used as the transaction cost parameter, whereas the right hand doesn't account for transaction costs, i.e., $\rho_c = 0$ in (4.11).

When transaction costs exist and are higher, the wealth performance is expected to be lower with a smaller rebalance interval δ because of the frequent rebalancing. This conjecture seems to hold when the problem with $\rho_c = 0$ is solved as in the right hand side of Fig. 5.2. For example, the Sharpe ratio drops significantly as δ goes to 1 in the case of $\rho = 0.5\%$. On the other hand, this tendency is not necessarily observed in the left hand side where the problem with $\rho_c = \rho$ is solved. Although the Sharpe ratio fluctuates as δ changes, and drops a little as δ moves closer to 1, there is not a clear dependence of the Sharpe ratio on δ for δ less than 20.

The effects of the GE constraint on the wealth performance are compared for the rebalance intervals $\delta = 1$ and $\delta = 10$ (with the same prediction horizon of $\tau = 40$) as shown in the left and right hand sides of Fig. 5.3, respectively. The upper lines in both sides indicate the relation between the Sharpe ratio and transaction cost

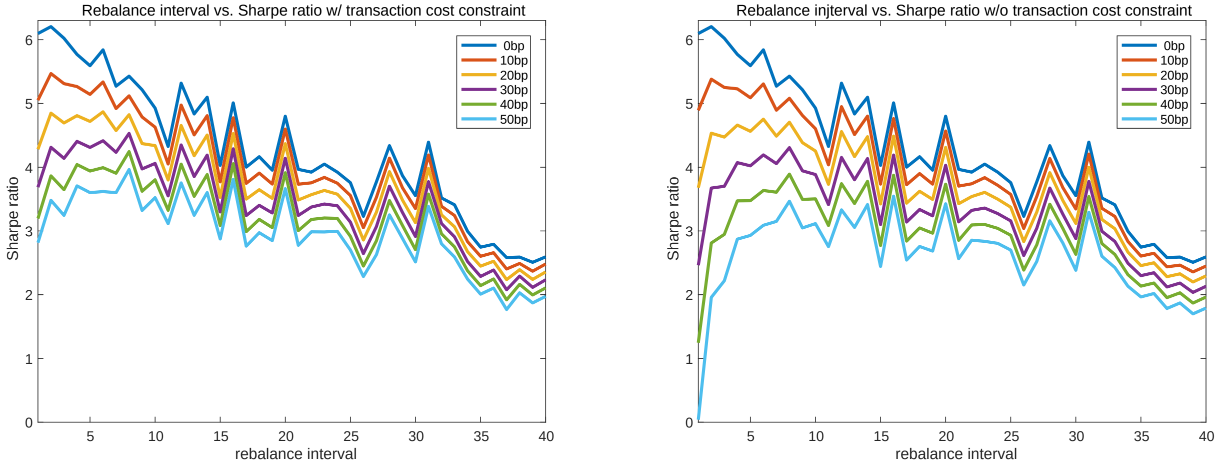

Fig. 5.2: Effect of rebalance interval on the wealth performance. The left hand side plots the Sharpe ratio for MPC with transaction cost constraint and the right hand side without transaction cost constraint, where the prediction horizon is set to $\tau = 40$.

rate when MPC w/ GE constraint is applied, whereas the lower lines show the ones w/o GE constraint. Note that the solid lines are obtained by solving (4.11) with the transaction cost parameter set to $\rho_c = \rho$ and the dashed lines are without it, i.e., $\rho_c = 0$. As already observed in Figs. 5.1 and 5.2, we see that the Sharpe ratio is improved by incorporating the transaction cost parameter $\rho_c = \rho$, but in fact, it can be further improved by adding the GE constraint. Although a similar tendency is observed for both sides in Fig. 5.3, the gap between w/ and w/o constraints seems to be larger for the right hand side than for the left hand side.

5.4 Comparison with unconditional MV optimal portfolio

Finally, it is of interest to compare our proposed MPC algorithm for pairs trading with the standard (unconditional) non-pairs based MV optimal portfolio, which is computed by applying the Markowitz (1952) model that allows short selling. This comparison allows us to understand the effectiveness of using spread information (i.e., the VAR model of cointegrated pairs) and a conditional MV approach as in the MPC algorithm for pairs trading, versus the unconditional MV objective used in the Markowitz model without spread information.

In the MV optimal portfolio, we estimate expected values and a covariance matrix of stock returns using the same stock price data (i.e., $27 \times 2 = 54$ stocks) and parameter estimation period as in the MPC algorithm, and maximize the expected total return of the portfolio minus the product of one-half the risk aversion coefficient γ and the variance (of total return). Here we explore different values of risk aversion coefficient γ and compare the portfolio performance between the MV optimal portfolio and MPC w/ and w/o GE constraint. Since the standard MV optimal portfolio can be considered myopic since it uses the expected value and variance of the 1 day portfolio return, to provide a fair comparison we set $\tau = \delta = 1$ in the MPC algorithm and consider the case of no transaction cost (and hence $\rho_c = 0$).

5.4.1 In-sample simulations

Fig. 5.4 illustrates the simulation results using in-sample data, where the term "in-sample" indicates that the portfolio performance is evaluated using the same data as in the parameter estimation period. The solid line

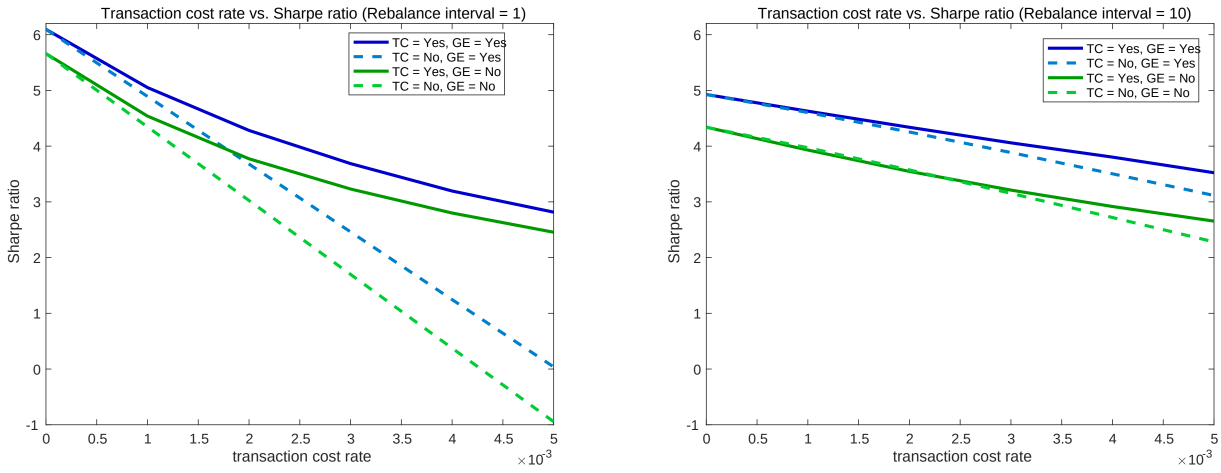

Fig. 5.3: Effects of GE constraint on the wealth performance are compared for rebalance intervals $\delta = 1$ (left) and $\delta = 10$ (right) with the prediction horizon of $\tau = 40$.

in the left hand side denotes the Sharpe ratio of MPC with GE constraint for different values of risk aversion coefficients, γ, whereas the dotted horizontal line represents the one without GE constraint. The Sharpe ratio obtained from the MV portfolio (i.e., Markowitz model) is also plotted as the dashed bottom line. From these plots, we first observe that the Sharpe ratio of the MPC approach is much higher than that of the MV portfolio. Also, we see that the Shape ratio of MPC w/ GE constraint is lower than that of MPC w/o GE constraint, in particular when γ is smaller, indicating that the GE constraint actually penalizes the optimality in terms of the Sharpe ratio. This effect weakens when γ becomes larger, and actually, the Sharpe ratio of MPC w/ GE constraint becomes higher than that w/o GE constraint and takes its maximum around $\gamma \simeq 6 \times 10^2$. This suggests that adjusting the GE constraint may improve the portfolio performance even in the case of in-sample simulations.

The solid and the dashed lines in the right hand plot of Fig. 5.4 compare the terminal wealth vs. the risk aversion coefficient γ for the MPC w/ GE constraint and the MV portfolio, respectively. From the right hand plot in Fig. 5.4, we see that the terminal wealth decreases with γ and that there is a clear relation between the choice of γ and the wealth performance. That is, the larger γ the larger the Sharpe ratio, but the lower the terminal wealth so that there is a trade-off between the profitability and the efficiency in terms of terminal wealth and Sharpe ratio, respectively.

Remark 1 *Note that the terminal wealth of MPC w/o GE constraint is not shown in the right hand side of Fig. 5.4, since the wealth process w/o GE constraint often provides steep up and down movement, which leads to unrealistic values of the wealth. Also, in several cases the wealth drops below zero indicating that the fund is in default in the case of out-of-sample simulation, although we omit the details. This is actually an important role of the GE constraint, where the fund may be able to avoid the risk of default by imposing the GE constraint.*

5.4.2 Out-of-sample simulations

We then execute an out-of-sample simulation, i.e., the portfolio performance is evaluated using the last one year period after the initial two year parameter estimation period. The left and right hand plots of Fig. 5.5 show the Sharpe ratios and the terminal wealth in the case of out-of-sample simulations for MPC and the MV portfolio, respectively. Compared with the in-sample simulation results in Fig. 5.4, the out-of-sample performance of both

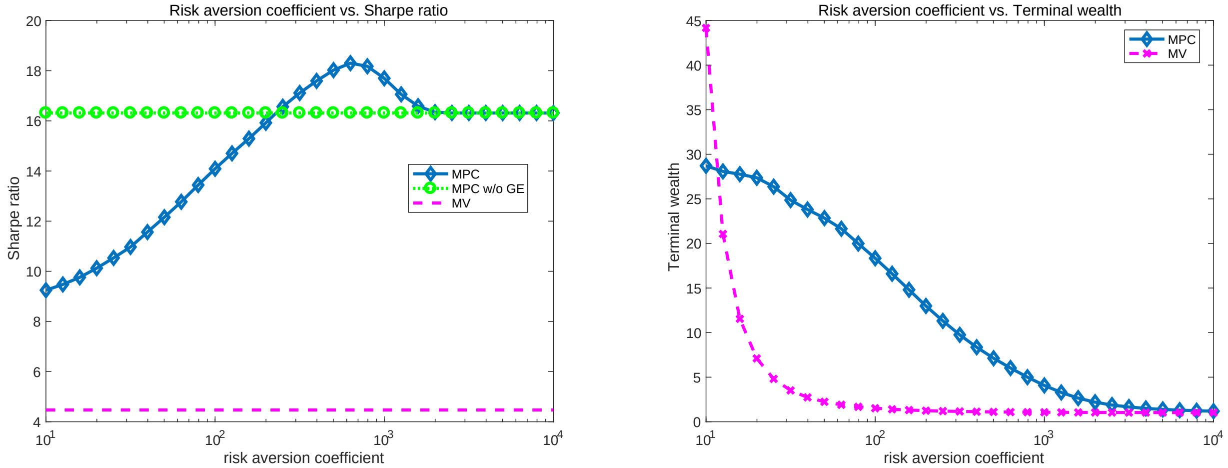

Fig. 5.4: In-sample simulations of MPC with $\delta = 1$ and $\tau = 1$ and the MV portfolio.

portfolios (i.e., the MPC and the MV portfolio) is significantly lower. In particular, the Shape ratio becomes negative and the terminal wealth is below the initial value in the case of the MV portfolio. On the other hand, the Sharpe ratio and the terminal wealth obtained by MPC are still at a reasonable level, even though they are not as good as those of the in-sample simulations. For example, the Sharpe ratio for MPC is always greater than 1 and the minimum increase in the terminal wealth is about 10% in one year.

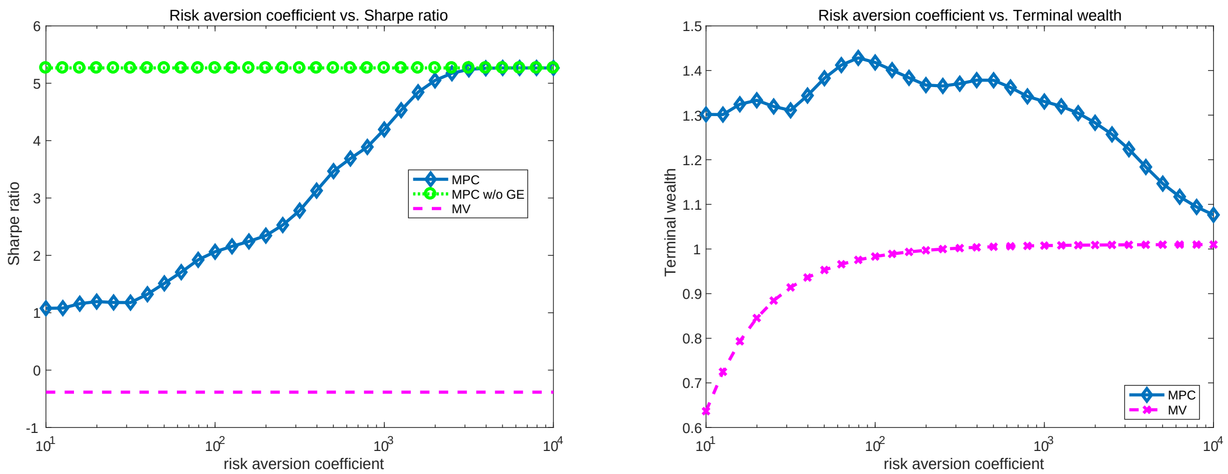

Fig. 5.5: Out-of-sample simulations of MPC with $\delta = 1$ and $\tau = 1$ and the MV portfolio.

6 Conclusion

In this paper, we have extended the MPC with linear feedback setting in Yamada and Primbs (2012) to address important issues related to transaction costs and GE constraints. First, we formulated a portfolio optimization problem involving cointegrated pairs of stocks following the result of Yamada and Primbs (2012), and developed an MPC strategy that calculates the conditional MV optimal portfolio for a given prediction horizon at each step. Then, we incorporated gross exposure and proportional transaction cost constraints in the conditional MV problem, where we showed that the problem reduces to a convex quadratic optimization even with these additional constraints. Based on numerical experiments using Japanese stock data, we demonstrated that solving the problem with transaction cost constraints improves the empirical performance of the wealth in terms of the Sharpe ratio, and is further improved by adding the GE constraint.

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A Pairs selection procedure and the list of selected pairs

We have selected 27 pairs using the procedure explained in Yamada and Primbs (2012) to perform empirical simulations in Section 5, which is briefly summarized as follows:

- Apply a screening procedure based on the Dickey-Fuller (DF) statistic[5] and correlation coefficient for each pair in stock universe, and sort the pairs that passed the screening procedure by DF statistic from smallest to largest, where smaller DF statistic indicates more significance.

- Select pairs from the top of the list down. Once a pair has been selected, remove all pairs further down in the list that contain either of the selected companies in the pair. Stop at a desired number of pairs or continue until the end of the list.

The selected pairs are listed in Table A.1, which have DF statistics smaller than the 1% critical value and the absolute values of correlation coefficients greater than $\sqrt{0.8}$. Note that an additional criterion that is sometimes used is to only select pairs where both stocks are in the same industry. We did not apply such a constraint, and as a result, we see that several pairs in the list are chosen from different industry categories.

[5] See Dickey and Fuller (1979).

Table A.1: List of selected pairs in the simulations

No.	Code	Company name	Code	Company name
1	9433	KDDI Corp.	4507	Shionogi Co., Ltd.
2	1803	Shimizu Corp.	4519	Chugai Pharmaceutical Co., Ltd.
3	6501	Hitachi, Ltd.	7205	Hino Motors, Ltd.
4	2502	Asahi Breweries, Ltd.	6762	TDK Corp.
5	7201	Nissan Motor Co.,	7272	Yamaha Motor Corp.
6	7211	Mitsubishi Motors Corp.	9766	Konami Corp.
7	8802	Mitsubishi Estate Co., Ltd.	7267	Honda Motor Co., Ltd.
8	9101	Nippon Yusen K.K.	7762	Citizen Holdings Co., Ltd.
9	1812	Kajima Corp.	9432	Nippon Telegraph and Telephone Corp.
10	8795	T&D Holdings, Inc.	8306	Mitsubishi UFJ Financial Group, Inc.
11	5714	Dowa Holdings Co., Ltd.	8355	The Shizuoka Bank, Ltd.
12	8411	Mizuho Financial Group, Inc.	8750	Dai-ichi Life Insurance Company, Limited
13	8766	Tokio Marine Holdings, Inc.	8630	Sompo Japan Nipponkoa Holdings, Inc.
14	1808	Haseko Corp.	8725	MS&AD Insurance Group, Inc.
15	2002	Nisshin Seifun Group Inc.	1925	Daiwa House Industry Co., Ltd.
16	9007	Odakyu Electric Railway Co., Ltd.	2802	Ajinomoto Co., Inc.
17	8252	Marui Group Co., Ltd.	1928	Sekisui House, Ltd.
18	4324	Dentsu Inc.	9020	East Japan Railway Company
19	4183	Mitsui Chemicals, Inc.	9602	Toho Co., Ltd.
20	4452	Kao Corp.	9735	Secom Co., Ltd.
21	3407	Asahi Kasei Corp.	3436	SUMCO Corp.
22	4901	Fujifilm Holdings Corp.	9202	All Nippon Airways Co., Ltd.
23	5108	Bridgestone Corp.	9062	Nippon Express Co., Ltd.
24	8804	Tokyo Tatemono Co., Ltd.	5401	Nippon Steel Corp.
25	5411	JFE Holdings, Inc.	5406	Kobe Steel, Ltd.
26	8015	Toyota Tsusho Corp.	6471	NSK Ltd.
27	1963	JGC Corporation	1801	Taisei Corp.