Franco-Japanese Days on Combinatorics and Optimization 2017 - In Honour of Michel Deza -

Date : December 4 (Mon.) – 5 (Tue.), 2017
Venue : University of Tsukuba, Tokyo Campus, Room 119
`http://www2.gssm.otsuka.tsukuba.ac.jp/staff/saru/FJ_2017.html`

Program

Day 1 (Monday, December 4)

12:00–12:55	Registration
12:55–13:00	Opening remarks, chaired by Makoto Namiki (Toho Univ.) Yasufumi Saruwatari (Univ. of Tsukuba) Komei Fukuda (ETH Zurich)
13:00–14:20	Session 1, chaired by Yasuko Matsui (Tokai Univ.) **Finding the boundary nodes of a Euclidean graph** Reinhardt Euler (Univ. of Brest) **Deza's last problems** Yohji Akama (Tohoku Univ.)
14:35–15:55	Session 2, chaired by Tomomi Matsui (Tokyo Inst. Tech.) **Optimizing illusion for ambiguous cylinders** Kokichi Sugihara (Meiji Univ.) **Cone of metrics, quasi-metrics and hemimetrics** Mathieu Dutour (Rudjer Boskovic Inst.)
16:10–17:30	Session 3, chaired by Akihisa Tamura (Keio Univ.) **Kempe equivalence of 3-edge-colorings in cubic graphs** Kenta Ozeki (Yokohama National Univ.) **On discrete midpoint convexity** Kazuo Murota (Tokyo Metropolitan Univ.)
18:00–20:00	Wine Party

Day 2 (Tuesday, December 5)

13:00–14:20 Session 4, chaired by Yoichi Izunaga (Univ. of Tsukuba)

On a conjecture of Michel Deza
David Avis (Kyoto Univ. and McGill Univ.)

Upper bounds on the diameter of polyhedra
Noriyoshi Sukegawa (Chuo Univ.)

14:35–15:55 Session 5, chaired by Tomomi Matsui (Tokyo Inst. Tech.)

A weighted linear matroid parity algorithm
Yusuke Kobayashi (Univ. of Tsukuba)

Principally box-integer polyhedra and equimodular matrices
Roland Grappe (Univ. Grenoble-Alpes and Univ. Paris 13)

16:00–16:40 Session 6, chaired by Tomomi Matsui (Tokyo Inst. Tech.)

Remembering Michel - Born free
Komei Fukuda (ETH Zurich)

16:40–16:45 Closing remarks, chaired by Makoto Namiki (Toho Univ.)

Chizuru Nishio (Univ. of Tsukuba)
Yasufumi Saruwatari (Univ. of Tsukuba)

Finding the boundary nodes of a Euclidean graph

Reinhardt Euler
University of Brest, France

Given a connected Euclidean graph G, we are looking for a simple polygon formed by edges of G such that all vertices of G are either on the polygon or surrounded by it. Applications include boundary detection of wireless sensor networks, monitoring of sensitive sites or clustering large data sets. To find such a polygon, we present a new algorithm called Least Polar-angle Connected Node (LPCN), whose basic idea is to construct a set of vertices $\{v(1), ..., v(h)\}$ in such a way that any pair of consecutive edges $\{v(i-1), v(i)\}$, $\{v(i), v(i+1)\}$ form a minimum polar angle. We also present D-LPCN, the distributed version of LPCN, and conclude with a discussion of simulation results and the implementation of both algorithms using real sensor nodes.

Deza's last problems

Yohji Akama
Tohoku University, Japan

We discuss some problems Deza raised in relation to metric geometry and theory of polycycles, in Sendai, Japan, on September, 2016.

1. Spherical monohedral polygonal tilings on the dual graphs of Table 9.1 of [1]. (M. Deza named spherical equi-cell tilings.)
2. Trigonometric approach to Helixene in relation to winding number.
3. Spherical trigonometry and Theorem 8.2.1 of [1, § 8.2, Non-extensible polycycles] (partial face-matching and angle-assignment).
4. Igor Revin's parametrization of Fullerens and Jacquemet study.
5. Right-angled hyperbolic polyhedra (Andrei Vesnin).
6. Two-sided bounds for the volume of right-angled hyperbolic polyhedra (D. Repovş and A. Vesnin).

References

[1] M.-M. Deza, M. Dutour Sikiríc. Geometry of Chemical Graphs: Polycycles and Twofaced Maps. Encyclopedia of Mathematics, Cambridge University Press, 2008.

[2] M.-M. Deza, M. Dutour Sikiríc, M. I. Shtogrin. Fullerene-like spheres with faces of negative curvature. pp. 251–274 in Diamond D5 and Related Nanostructures Carbon Materials: Chemistry and Physics, Volume 6, ed. by M.V. Diudea and C.L. Nagy, Springer, 2013.

[3] M.-M. Deza, M. Dutour Sikiríc, M. I. Shtogrin. Geometric Structure of Chemistry-Relevant Graphs. Zigzags and Central Circuits. Forum for Interdisciplinary Mathematics 1, Springer, 2015.

Optimizing illusion for ambiguous cylinders

Kokichi Sugihara
Meiji University, Japan

We present a method for designing a class of 3D objects called “ambiguous cylinders,” which create new visual illusion. Ambiguous cylinders are objects whose shapes change drastically when they are reflected in a mirror so that it is almost unbelievable that they come from the same objects. In this sense they belong to impossible objects. Creation of ambiguous cylinders are based on two observations, one is mathematical and the other is psychological. Mathematically a single image does not convey depth information and hence there are infinitely many different 3D shapes that generate the same image. Psychologically, on the other hand, our brains prefer right angles in interpreting 2D images as 3D shapes. Combining these two observations, we can design cylinder-like objects that give two desired appearances, one in the direct view and the other in the mirror. There are also variants of ambiguous cylinders. They are “partly invisible objects,” whose parts become invisible in the mirror, “topology-disturbing objects,” whose topology changes in the mirror, and “deformable objects,” whose shapes appear to change smoothly when we change the view direction continuously. All of those behaviors are physically impossible, but appear to occur because of visual illusion. We present a necessary and sufficient condition for realizing the 3D object that generates a given pair of 2D appearances, and a computational method for designing such a 3D object. These arguments also tell us that for a given pair of 2D appearances that satisfies the necessary and sufficient condition, the associated ambiguous cylinder is unique.

Next we consider how to optimize illusion, i.e., how to maximize the sense of impossibility. For that purpose we have to consider several aspects. First, each one of us has two eyes and can combine them to understand the true shape of the 3D object. So some impossible objects work only when they are seen by a single eye. Second shading information gives clues about the true shape of the 3D objects. So some impossible objects work only when the illumination conditions are carefully tuned. Thirdly, change of the view point gives many images of the same object, and help us to understand the true shape. So some of the impossible objects work only when the view position is fixed. Considering these aspects, the maximum sense of impossibility might be attained if we can create a 3D object that appears impossible even if we use two eyes, we move our head, and we put the object in a usual illumination environment. We found that the class of the “deformable objects” fulfill all those requirements, because the illusion occurs when we see them by our two eyes in an ordinary illumination environment, and rotate the objects by our hands around the vertical axis.

Cone of metrics, quasi-metrics and hemimetrics

Michel Deza — École Normale Supérieure
Mathieu Dutour — Rudjer Boskovic Institute, Croatia

In this talk, I will present the works that we did together on the subject of metric cones and related subjects. Given a finite point set $X = \{1, \ldots, n\}$, we can define the cone of metric on this point set X. That is the set of function $d : X \times X \mapsto \mathbb{R}$ such that

- $d(x, x) = 0$ for all $x \in X$
- $d(x, y) = d(y, x)$ for all $x, y \in X$
- $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$

is a polyhedral cone that we call $\text{MET}(K_n)$. A particular very interesting subset of this cone is the cone of L^1 embeddable metrics, that we call $\text{CUT}(K_n)$. The vertices/facets of those cones are known up to $n = 8$.

We shortly present the algorithms that were used for the computation of the facets of polytopes given by their vertices. The programs are freely available on the second author web page. Some applications to the Bell polytopes are presented. We also shortly present the hypermetrics.

The definition of the metric and cut cone can be extended to an arbitrary graph G. The triangle inequalities are replaced by cycle inequalities and non-negative inequalities. In that setting we have $\text{CUT}(G) = \text{MET}(G)$ is and only if G does not have a K_5 minor. This allows to compute the facets of many cut polytope and is a remarkable result.

One natural generalization of metric is to consider the cone of quasimetrics defined as functions $d : X \times X \mapsto \mathbb{R}$ such that

- $d(x, x) = 0$ for all $x \in X$,
- $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

In that setting we define a notion of metric polytope of a graph that we call $\text{QMET}(G)$ and we give an explicit set of inequalities describing it that generalizes the one for $\text{MET}(G)$. We define the notion of oriented metrics that are weightable and an oriented version of the cuts.

Another generalization is to consider the notion of metrics on more than 2 points, i.e. hemimetrics. In that setting the equivalent of the triangle inequality would be the inequality over a simplex. However, it turns out that this definition is not workable since it does not allow to define the hemimetrics on a simplicial complex. We give another set of inequalities that allow a neat generalization to the case of an arbitrary complex.

Kempe equivalence of 3-edge-colorings in cubic graphs

Kenta Ozeki
Yokohama National University, Japan

Let G be a cubic graph having a 3-edge-coloring. For a 2-edge-colored cycle D, a *Kempe switch (at D)* is an operation to obtain another 3-edge-coloring by switching the colors of $E(D)$. Two 3-edge-colorings in G are *Kempe equivalent* if one is obtained from the other by the sequence of Kempe switches. Kempe equivalence is a well-known method to study cubic graphs, for example, the 4 Color Theorem, the 4-flow conjecture, and so on.

Mohar asked which cubic graphs have only one Kempe equivalence class. Fisk proved that every bipartite planar graph has such a property. In this paper, we consider the projective planar case, showing that a bipartite cubic graph G on the projective-plane admits only one Kempe equivalent class if and only if the dual G^* is not 4-vertex-colorable. In order to prove this theorem, we use the *signature* of 3-edge-colorings. As a byproduct, we also show that every 3-edge-colorable graph G on the projective plane is 3-list-edge-colorable if the dual G^* is not 4-vertex-colorable.

On discrete midpoint convexity

Kazuo Murota
with Satoko Moriguchi, Akihisa Tamura, and Fabio Tardella
Tokyo Metropolitan University, Japan

For a function defined on a convex set in a Euclidean space, midpoint convexity is the property requiring that the value of the function at the midpoint of any line segment is not greater than the average of its values at the endpoints of the line segment. Midpoint convexity is a well-known characterization of ordinary convexity under very mild assumptions. For a function defined on the integer lattice, we consider the analogous notion of discrete midpoint convexity, a discrete version of midpoint convexity where the value of the function at the (possibly noninteger) midpoint is replaced by the average of the function values at the integer round-up and round-down of the midpoint. It is known that discrete midpoint convexity on all line segments with integer endpoints characterizes $L^\natural$-convexity, and that it characterizes submodularity if we restrict the endpoints of the line segments to be at ℓ_∞-distance one. By considering discrete midpoint convexity for all pairs at ℓ_∞-distance equal to two or not smaller than two, we identify new classes of discrete convex functions, called locally and globally discrete midpoint convex functions, which are strictly between the classes of $L^\natural$-convex and integrally convex functions, and are shown to be stable under scaling and addition. Furthermore, a proximity theorem, with the same small proximity bound as that for $L^\natural$-convex functions, is established for discrete midpoint convex functions. Relevant examples of classes of locally and globally discrete midpoint convex functions are provided.

Keywords: Midpoint convexity, Discrete convex function, Integrally convex function, $L^\natural$-convex function, Proximity theorem, Scaling operation

On a conjecture of Michel Deza

David Avis
Kyoto University and McGill University, Canada

Ron Graham once told me that most people find it hard to abandon the topic of their PhD thesis and I am no exception. In 1960, writing in Russian as M.E. Tylkin, Deza introduced what is now called the cut cone - the conic hull of the edge incidence vectors of cuts in the complete graph K_n. He showed for $n = 4$ its facets are the triangle inequalities. In 1972, writing in French as M. Deza, he continued this study for larger n describing a general class of inequalities now known as hypermetric inequalities. He proved that the simplest of these, the triangle and pentagonal inequalities, described completely the cut cone for $n = 5$. He asked whether the hypermetric inequalities generated the facets of the cut cone for all n. Vasek Chvátal explained this problem to me in 1974 and it became the starting point for my PhD thesis. I found subsequently that the answer was no. At the time I found it an interesting but rather narrow topic and remember being asked during my defense whether there were any applications. At the time I did not know of any. This talk will survey the development of what has turned out to be a very rich topic indeed. Applications have been found in embedability, mining, statistical physics, optimization, probability theory, quantum information theory and, most recently, quantum gravity and wormholes. I will discuss some of these applications as well as theoretical developments and end with some open problems.

Upper bounds on the diameter of polyhedra

Noriyoshi Sukegawa
Chuo University, Japan

Finding a good upper bound on the diameter of a polyhedron is an outstanding open problem in polyhedral combinatorics and mathematical optimization. Most notably, this problem is closely related to and motivated by the complexity analyses for the simplex method. In this context, a major open question is the polynomial Hirsch conjecture stating that the diameter is bounded from above by a polynomial in d and n, where d is the dimension and n is the number of facets of the given polyhedron. If this conjecture fails, then we shall conclude that the simplex method cannot be a polynomial-time algorithm.

In 1992, Kalai and Kleiman proved the first subexponential upper bound. Their proof is a consequence of a recursive inequality, which we call the Kalai-Kleitman inequality. In 2014, Todd gave a tighter analysis to the Kalai-Kleitman inequality to show a slightly improved upper bound. Although tight in low dimensions, Todd's bound can be improved further in high dimensions as demonstrated by the subsequent studies.

In this talk, we first show a tighter analysis to the Kalai-Kleitman inequality to yield an upper bound of $(n-d)^{\log \mathrm{O}(d/\log d)}$. The current best upper bound is $(n-d)^{\log \mathrm{O}(d)}$ and thus our result yields the first asymptotic improvement in the exponent (although it is still far from the conjectured bound). We then observe several limitations and properties of the Kalai-Kleitman inequality. In addition, we would like to mention alternative approaches for understanding the behavior of the diameter including the recent studies on the diameter of lattice polytopes.

A weighted linear matroid parity algorithm

Yusuke Kobayashi
University of Tsukuba, Japan

The matroid parity (or matroid matching) problem, introduced as a common generalization of matching and matroid intersection problems, is so general that it requires an exponential number of oracle calls. Lovasz (1980) showed that this problem admits a min-max formula and a polynomial algorithm for linearly represented matroids. Since then efficient algorithms have been developed for the linear matroid parity problem. We present a combinatorial, deterministic, polynomial-time algorithm for the weighted linear matroid parity problem. The algorithm builds on a polynomial matrix formulation using Pfaffian and adopts a primal-dual approach based on the augmenting path algorithm of Gabow and Stallmann (1986) for the unweighted problem. This is joint work with Satoru Iwata.

Principally box-integer polyhedra and equimodular matrices

Roland Grappe

University Grenoble-Alpes and University Paris 13, France

A polyhedron is box-integer if its intersection with any integer box is an integer polyhedron. We introduce principally box-integer polyhedra: they are the polyhedra whose dilatations are box-integer as soon as they are integer. We will present several characterizations of principally box-integer polyhedra, which involve matrices and strong integrality properties of linear systems. Eventually, we will discuss connections with combinatorial optimization problems, and more precisely about box-total dual integrality.

This is a joint work with Patrick Chervet (Lycee Olympe de Gouges) and Louis-Hadrien Robert (Universitat Hamburg).

Remembering Michel - Born free

Komei Fukuda
ETH Zurich, Switzerland

The first time I met Michel Deza was at an AMS (American Mathematics Society) meeting in the US, late 1970's, when I was working for a PhD at University of Waterloo, Canada. He gave a talk, extremely passionate, loud, and "wild" in broken English. It left me a strong impression of a life of free mathematician that does not exist in Japan, and I found it fascinating.

I did not have a chance to work with Michel while I was a doctoral student in Canada (1976-81). The first chance came in the mid 1980's when I was working at Tokyo Institute of Technology. He was a visitor to the laboratory of Masakazu Kojima where I was a baby researcher. We were both interested in set families and matroids. A natural subject emerged and we studied certain combinatorial operations on set families. Michel was obsessively interested in computational experiments and could spend many hours just looking at experiments that I printed out for him. He was exceptionally sharp to recognize non-trivial phenomena that were revealed by clouds of numbers. Michel had a unusual instinct of digging out something beautiful. I was very lucky to be able to work with such a pure, rare and passionate mathematician.

Michel was obviously fond of Japanese life. Probably it served as a place of calmness and serenity. Even for Michel who enjoyed a free world of severe competitions and glories, he valued the country of order and politeness. Yet, he knew that living in a very cosy place too long can lead to a life too protected and not exciting. That is the reason, as I believe, why Michel suggested me to leave again for the western world, in particular, for Paris where he worked. I guess that Michel saw that living too long in Japan, I started to look like a tamed animal. Following his advise I left for Paris for nine months that changed my life forever. Michel's experiment to release a tamed animal to the wild had many consequences in the years to come. The Franco-Japanese conferences starting in 1988 made many chances for Japanese researchers to communicate with French and other foreign researchers. My eventual departure to Switzerland is another consequence that changed my life and family. Thanks Michel for teaching me how to live a free life.