Generalized Quantifiers in Dependent Type Semantics

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GQs in DTS

- Dependent Type Semantics (DTS) can explain various phenomena which involve anaphora and presupposition.
- How GQs can be analyzed in DTS?
  (Σ and Π types correspond to "some" and "every")
- It is still a challenging task to integrate analysis of classical GQs with type theoretical framework.
- Sundholm (1989) proposed a way to represent GQs based on dependent type theory.

Two approaches to GQs

(1) Constructing a proof system that contains GQs as primitives

- natural logic (Moss 2010)
- natural deduction system (Francez and Ben-Avi 2014)

Currently, however, such a proof system is not concerned with dynamic linguistic phenomena.

(2) Representing meanings of GQs explicitly

- Sundholm's (1989) analysis based on *Dependent Type Theory* (DTT)

We adopt this second explicit approach and extend it to account for dynamic behaviors of GQs

Outline

- Sundholm's constructive GQs (MOST)
  - Basic idea
  - Proportion problem
- GQs in DTS
  - Donkey anaphora
  - Strong/weak reading
  - Inter-sentential anaphora

Sundholm (1989): Constructive GQs

Sundholm's constructive GQs

- Finately many, countably many, most, there are more ... than ..., ...

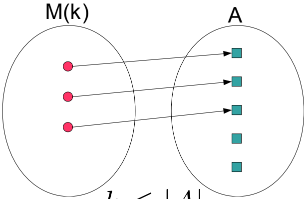
(Classical) meaning of $\textsc{most}(A, B)$:

1. $|A \wedge B| \geq |A \wedge \neg B|$
2. $|A \wedge B| \geq \frac{|A|}{2}$

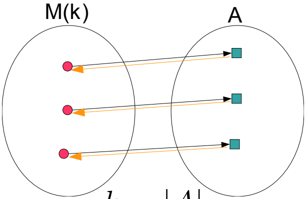
The second formalization enable us to handle other quantifiers in a same manner.

Basic idea: Cardinality

- To count the cardinality of "set" A, we need:
  (1)$M(k)$ (2)mapping
- $M(k)$ is a "set" whose cardinality is $k : N$.
- Counting the elements in "set" A is implemented via mapping $f : M(k) \to A$
  - There are at least k elements in A
    $\equiv$ There is an injection from $M(k)$ to A
  - There are exactly k elements in A
    $\equiv$ There is a bijection from $M(k)$ to A



$k \leq |A|$



$k = |A|$

Injection and Surjection

Injection

$$\frac{A : \text{type} \qquad B : \text{type} \qquad f : A \to B}{\textbf{Injection}(f) \stackrel{def}{\equiv} (y{:}A) \to (y'{:}A) \to (fy =_B fy') \to (y =_A y')}$$

Surjection

$$\frac{A : \text{type} \qquad B : \text{type} \qquad f : A \to B}{\textbf{Surjection}(f) \stackrel{def}{\equiv} (z{:}B) \to \begin{bmatrix} y{:}\, A \\ z =_B fy \end{bmatrix}}$$

Bijection and Finite

Bijection

$$\frac{A : \textsf{type} \qquad B : \textsf{type} \qquad f : A \to B}{\textbf{Bijection}(f) \overset{def}{\equiv} \begin{bmatrix} \textbf{Injection}(f) \\ \textbf{Surjection}(f) \end{bmatrix}}$$

Finite

$$\frac{A : \textsf{type}}{\textbf{Finite}(A) \overset{def}{\equiv} \begin{bmatrix} k' : N \\ \begin{bmatrix} f : M(k') \to A \\ \textbf{Bijection}(f) \end{bmatrix} \end{bmatrix}}$$

Note: proof-term of **Finite**

Finite (previously shown)

$$\frac{A : \text{type}}{\textbf{Finite}(A) \stackrel{def}{\equiv} \begin{bmatrix} k' : N \\ \begin{bmatrix} f : M(k') \to A \\ \textbf{Bijection}(f) \end{bmatrix} \end{bmatrix}}$$

Given A, a term of type **Finite**(A) has a structure of tuple.

$(n, (f, b))$

a : **Finite**(A)

$\pi_1\, a$ ··· natural number corresponds to A's cardinality

$\pi_1 \pi_2\, a$ ··· function f

$\pi_2 \pi_2\, a$ ··· proof of f being a bijection

Sundholm's MOST

$$\frac{A : \mathsf{type} \quad B : A \to \mathsf{type} \quad a : \textbf{Finite}(A)}{\textbf{Most}(A, B) \stackrel{def}{\equiv} \begin{bmatrix} k : N \\ \begin{bmatrix} k \geq [\frac{\pi_1(a)}{2}] + 1 \\ \begin{bmatrix} f : M(k) \to A \\ \begin{bmatrix} \textbf{Injection}(f) \\ (y : M(k)) \to B(fy) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}}$$

Proportion problem

Most farmers who own a donkey beat it.

$$\mathbf{Most}\left(\begin{bmatrix} x\colon \mathbf{Farmer} \\ \begin{bmatrix} y\colon \mathbf{Donkey} \\ \mathbf{Own}(x,y) \end{bmatrix} \end{bmatrix}, \lambda z.\mathbf{Beat}(z)\right)$$

- Proof-terms to be counted has type

$$\begin{bmatrix} x\colon \mathbf{Farmer} \\ \begin{bmatrix} y\colon \mathbf{Donkey} \\ \mathbf{Own}(x,y) \end{bmatrix} \end{bmatrix}.$$

  Thus, the terms have a structure of tuple $(x,\ (y,\ p))$, where $x : \mathbf{Farmer}, y : \mathbf{Donkey}, p : \mathbf{Own}(x,y)$

- Counting such terms causes the so-called proportion problem (Kadmon 1990)

Proportion problem (1)

Counting pairs leads to unintended quantification:

> *Suppose that there are ten farmers. Nine of them own one donkey each and they do not beat their donkies. The tenth farmer owns 45 donkies and beats all of them.*

45 tuples out of 54 meet beating relation.

Proportion problem (2)

- There might be many proofs for a proposition
  - a proof of "There is a man"

$$\begin{array}{c} \text{john} : \textbf{entity} \\ m_j : \textbf{Man}(\text{john}) \\ \downarrow \\ (\text{john}, m_j) : \begin{bmatrix} x : \textbf{entity} \\ \textbf{Man}(x) \end{bmatrix} \end{array} \qquad \begin{array}{c} \text{bill} : \textbf{entity} \\ m_b : \textbf{Man}(\text{bill}) \\ \downarrow \\ (\text{bill}, m_b) : \begin{bmatrix} x : \textbf{entity} \\ \textbf{Man}(x) \end{bmatrix} \end{array}$$

  - proofs which are obtained via different inference

- Sundholm suggested to assume that a proposition has only-one proof, and others are reducible to the proof.
- Another possibility: assume proof irrelevance (Luo 2012)
- In DTS, we allow different proofs for one proposition.

Solution to the proportion problem

$$(x, (y, p)) : \begin{bmatrix} x : \textbf{Farmer} \\ \begin{bmatrix} y : \textbf{Donkey} \\ \textbf{Own}(x, y) \end{bmatrix} \end{bmatrix}.$$

Sundholm's proposal: counting the elements in set **Farmer**
$\rightsquigarrow$ Define an injection which is relative to set B in the restrictor $\begin{bmatrix} x : B \\ C(x) \end{bmatrix}$ (**Injection**$_B$).

Definition of **Injection**$_B$

Injection (previously shown)

$$\frac{A : \mathsf{type} \qquad B : \mathsf{type} \qquad f : A \to B}{\textbf{Injection}(f) \overset{def}{\equiv} (y{:}A) \to (y'{:}A) \to (fy =_B fy') \to (y =_A y')}$$

Injection$_B$

$$\frac{A,\ B : \mathsf{type} \qquad C : B \to \mathsf{type} \qquad f : A \to \begin{bmatrix} x{:}\ B \\ C(x) \end{bmatrix}}{\textbf{Injection}_B(f) \overset{def}{\equiv} (y{:}A) \to (y'{:}A) \to (\pi_1(fy) =_B \pi_1(fy')) \to (y =_A y')}$$

When the first projection of fy and fy' are same, those terms will be not counted twice.

Definition of Surjection$_B$

Surjection (previously shown)

$$\frac{A : \text{type} \qquad B : \text{type} \qquad f : A \to B}{\textbf{Surjection}(f) \overset{def}{\equiv} (z{:}B) \to \begin{bmatrix} y{:}\, A \\ z =_B fy \end{bmatrix} : \text{type}}$$

Surjection$_B$

$$\frac{A,\ B : \text{type} \qquad C : B \to \text{type} \qquad f : A \to \begin{bmatrix} x{:}\, B \\ C(x) \end{bmatrix}}{\textbf{Surjection}_B(f) \overset{def}{\equiv} \left(z{:} \begin{bmatrix} x{:}\, B \\ C(x) \end{bmatrix}\right) \to \begin{bmatrix} y{:}\, A \\ \pi_1 z =_B \pi_1(fy) \end{bmatrix} : \text{type}}$$

Not every $z : \begin{bmatrix} x{:}\, B \\ C(x) \end{bmatrix}$ has its counterpart $y : A$ which is directly mapped to z by f

Revised version

$$\frac{A : \text{type} \quad B : A \to \text{type} \quad C : \begin{bmatrix} x\colon A \\ B(x) \end{bmatrix} \to \text{type} \quad a : \textbf{Finite}_A(\begin{bmatrix} x\colon A \\ B(x) \end{bmatrix})}{\text{MOST}(\begin{bmatrix} x\colon A \\ B(x) \end{bmatrix}, C) \stackrel{def}{\equiv} \begin{bmatrix} k\colon N \\ \begin{bmatrix} k \geq [\frac{\pi_1(a)}{2}] + 1 \\ \begin{bmatrix} f\colon M(k) \to \begin{bmatrix} x\colon A \\ B(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_A(f) \\ (y{:}M(k)) \to C(fy) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}}$$

Phenomena

How this approach works when GQs interacts with anaphora?

⋆ Resolution of donkey anaphora

(1) Most farmers who own a donkey beat it.

⋆ Inter-sentential anaphora

(2) Three students wrote a paper. #He also read a book.

(3) Three students wrote a paper. They also read a book.

(4) Three students wrote a paper. They weren't very good.

(5) Three students wrote a paper. They sent it to L&P.

GQs in DTS (1)
GQs and intra-sentential anaphora

Common nouns: types or predicates?

Common nouns as types: Ranta (1994), Luo (2012a, b)

(6) A man walks. $\begin{bmatrix} x : \textbf{man} \\ \textbf{walk}(x) \end{bmatrix}$

Common nouns: types or predicates?

Common nouns as types: Ranta (1994), Luo (2012a, b)

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The problem of predicate nominals

(7) John is a <u>man</u>. (predicational sentence)

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(6) A man walks. $\begin{bmatrix} x\colon \textbf{man} \\ \textbf{walk}(x) \end{bmatrix}$

The problem of predicate nominals

(7) John is a man. (predicational sentence)

1. Predicational sentences stand for judgements?

1. John is a man.	**john** : **man**
2. John is not a man.	**??**
3. If John is a man, then ...	**??**

- Judgement **john** : **man** cannot be negated nor appear in the antecedent of a conditional.

Common nouns: types or predicates?

2. The Russell-Montague's analysis?

1. John is a man. $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix}$
2. John is not a man. $\neg \begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix}$
3. If John is a man, then ... $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix} \rightarrow \ldots$

Common nouns: types or predicates?

2. The Russell-Montague's analysis?

1. John is a man. $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix}$
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3. If John is a man, then ... $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix} \rightarrow \ldots$

Problem 1

(8) # John is a man_1. He_1 is .... (Kuno 1970; Mikkelsen 2004)

$$\begin{bmatrix} u : \begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix} \\ \ldots \pi_1 u \cdots \end{bmatrix}$$

- A predicate nominal does not introduce a discourse

Common nouns: types or predicates?

2. The Russell-Montague's analysis?

1. John is a man. $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix}$
2. John is not a man. $\neg \begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix}$
3. If John is a man, then ... $\begin{bmatrix} x : \textbf{man} \\ \textbf{john} =_{\textbf{man}} x \end{bmatrix} \rightarrow \ldots$

Problem 2

- $\textbf{john} =_{\textbf{man}} x$ is well-formed only if **john** : **man** is provable.

$$\frac{A : \text{type} \quad t : A \quad u : A}{t =_A u : \text{type}} =F$$

- So, 2. and 3. presuppose that John is a man!
- Zhaohui Luo (p.c.) proposed a way out of this problem.

Common nouns: types or predicates?

DTS: Common nouns as predicates

(8) A man walks. $\begin{bmatrix} u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{man}\,(x) \end{bmatrix} \\ \textbf{walk}\,(\pi_1 u) \end{bmatrix}$

(9) John is a man. $\textbf{man}(\textbf{john})$

Definition of MOST_{wk}

$$\frac{A,\ B : \textbf{entity} \to \text{type} \qquad a : \textbf{Finite}_{\textbf{E}}\left(\begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix}\right)}{\text{MOST}_{wk}(A,B) \stackrel{def}{\equiv} \begin{bmatrix} k\colon N \\ \begin{bmatrix} k \geq [\frac{\pi_1(a)}{2}] + 1 \\ \begin{bmatrix} f\colon M(k) \to \begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_{\textbf{E}}(f) \\ \left(z\colon \begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix}\right) \to \\ \left(B(\pi_1 z) \leftrightarrow \begin{bmatrix} y\colon M(k) \\ z =_{\begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix}} fy \end{bmatrix}\right) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}}$$

The difference

Sundholm's original version:

$$(y{:}M(k)) \to B(\pi_1(fy))$$

Current analysis: the natural number k is an exact number of elements which satisfy both A and B

$$\left(z{:}\begin{bmatrix} x{:}\,\textbf{entity} \\ A(x) \end{bmatrix}\right) \to \left(\begin{bmatrix} y{:}\,M(k) \\ z = \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x) \end{bmatrix} fy \end{bmatrix} \to B(\pi_1 z)\right)$$

$$\left(z{:}\begin{bmatrix} x{:}\,\textbf{entity} \\ A(x) \end{bmatrix}\right) \to \left(B(\pi_1 z) \to \begin{bmatrix} y{:}\,M(k) \\ z = \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x) \end{bmatrix} fy \end{bmatrix}\right)$$

Weak and Strong reading

⋆ Two readings of the donkey sentence (Chierchia 1992):

(10) Most farmers who own a donkey beat it.

Weak reading

Most farmers who own a donkey beat some donkey they own.

Strong reading

Most farmers who own a donkey beat every donkey they own.

Weak and Strong reading

⋆ Two readings of the donkey sentence (Chierchia 1992):

(10) Most farmers who own a donkey beat it.

Weak reading

Most farmers who own a donkey beat some donkey they own.

Strong reading

Most farmers who own a donkey beat every donkey they own.

⋆ Sundholm's definition and MOST_{wk} predict weak reading.

$$\ldots \left(z: \begin{bmatrix} x: \textbf{entity} \\ A(x) \end{bmatrix}\right) \to \left(B(\pi_1 z) \leftrightarrow \begin{bmatrix} y: M(k) \\ z = \begin{bmatrix} x: \textbf{entity} \\ A(x) \end{bmatrix} fy \end{bmatrix}\right) \ldots$$

Definition of MOST_{str}

$$
\frac{A,\ B : \textbf{entity} \to \mathsf{type} \qquad a : \textbf{Finite}_{\mathbf{E}}\left(\begin{bmatrix} x \colon \textbf{entity} \\ A(x) \end{bmatrix}\right)}{\mathrm{MOST}_{str}(A, B) \stackrel{def}{\equiv} \begin{bmatrix} k \colon N \\ \begin{bmatrix} k \geq [\frac{\pi_1(a)}{2}] + 1 \\ \begin{bmatrix} f \colon M(k) \to \begin{bmatrix} x \colon \textbf{entity} \\ A(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_{\mathbf{E}}(f) \\ \left(z \colon \begin{bmatrix} x \colon \textbf{entity} \\ A(x) \end{bmatrix}\right) \to \\ \quad \left(B(\pi_1 z) \leftrightarrow \begin{bmatrix} y \colon M(k) \\ \pi_1 z =_{\textbf{entity}} \pi_1(fy) \end{bmatrix}\right) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}}
$$

Anaphora resolution

(11) Most farmers who own a donkey beat it.

$$
\left[\begin{array}{l}
k \colon N \\
\left[\begin{array}{l}
k \geq \left[\frac{\pi_1(a)}{2}\right] + 1 \\
\left[\begin{array}{l}
f \colon M(k) \to \left[\begin{array}{l} x \colon \textbf{entity} \\ \mathcal{A}(x) \end{array}\right] \\
\left[\begin{array}{l}
\textbf{Injection}_{\textbf{E}}(f) \\
\left(z \colon \left[\begin{array}{l}
x \colon \textbf{entity} \\
\left[\begin{array}{l}
\textbf{Farmer}(x) \\
\left[\begin{array}{l}
y \colon \textbf{entity} \\
\left[\begin{array}{l} \textbf{Donkey}(y) \\ \textbf{Own}(x, y) \end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}\right] \right) \to \\
\qquad \left(\textbf{Beat}(\pi_1 z, @ : \textbf{entity}) \leftrightarrow \left[\begin{array}{l}
y \colon M(k) \\
z = \left[\begin{array}{l} x \colon \textbf{entity} \\ \mathcal{A}(x) \end{array}\right] f y
\end{array}\right] \right)
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}\right]
$$

Extends to numerical quantifier

$$\frac{A,\ B : \textbf{entity} \to \mathsf{type} \qquad a : \textbf{Finite}_{\textbf{E}}\left(\begin{bmatrix} x : \textbf{entity} \\ A(x) \end{bmatrix}\right)}{\textsc{Three}(A, B) \stackrel{def}{\equiv} \begin{bmatrix} k : N \\ \begin{bmatrix} k =_N 3 \\ \begin{bmatrix} f : M(k) \to \begin{bmatrix} x : \textbf{entity} \\ A(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_{\textbf{E}}(f) \\ \left(z : \begin{bmatrix} x : \textbf{entity} \\ A(x) \end{bmatrix}\right) \to \\ \quad \begin{bmatrix} y : M(k) \\ z =_{\begin{bmatrix} x : \textbf{entity} \\ A(x) \end{bmatrix}} fy \end{bmatrix} \to B(\pi_1 z) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}}$$

GQs in DTS (2)
Inter-sentential anaphora

GQs and inter-sentential anaphora

(12) Three students wrote a paper. #He also read a book.

(13) Three students wrote a paper. They also read a book.

(14) Three students wrote a paper. They weren't very good.

(15) Three students wrote a paper. They sent it to L&P.

Distributive reading: *they*

(16) Three students wrote a paper. They sent it to L&P.

Krifka's (1996) parametrized sum individuals:

- Each student sent a paper he/she wrote.
- Each of those students, we need assignment function which is associated with that student.

Distributive reading: *they*

- In type theoretical framework, variables introduced in a context can have any sets as their types.
- We can have objects of function types in a context, which can be used afterwards (as mentioned in Ranta (1994)).

$$
(k,(P_k,(f,(I_f,\mathsf{a})))) : \begin{bmatrix} k : N \\ \begin{bmatrix} k =_N 3 \\ \begin{bmatrix} f : M(k) \to \begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_{\textbf{E}}(f) \\ \left(z : \begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} \right) \to \\ \qquad \begin{bmatrix} y : M(k) \\ z = \begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} fy \end{bmatrix} \to \begin{bmatrix} q : \begin{bmatrix} p : \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(\pi_1 z, \pi_1 q) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}
$$

Distributive reading: *they*

$$\text{let } t = @ : \begin{bmatrix} k \colon N \\ \begin{bmatrix} A \colon \textbf{entity} \to \text{type} \\ M(k) \to \begin{bmatrix} x \colon \textbf{entity} \\ A(x) \end{bmatrix} \end{bmatrix} \end{bmatrix} \text{ in}$$

$$\lambda P. \left(u \colon \begin{bmatrix} x \colon \textbf{entity} \\ (\pi_1 \pi_2 t)(x) \end{bmatrix} \right) \to \left(v \colon \begin{bmatrix} m \colon M(\pi_1 t) \\ u =_{\begin{bmatrix} x \colon \textbf{entity} \\ (\pi_1 \pi_2 t)(x) \end{bmatrix}} (\pi_2 \pi_2 t)(m) \end{bmatrix} \right) \to P(\pi_1 u)$$

Demonstration

Three students wrote a paper. They sent it to L&P.

Represented by using progressive conjunction:

$$\begin{bmatrix} c\colon \text{THREE}\left(\lambda x.\mathbf{Student}(x),\ \lambda x.\begin{bmatrix} q\colon \begin{bmatrix} p\colon \mathbf{entity} \\ \mathbf{Paper}(p) \end{bmatrix} \\ \mathbf{Write}(x, \pi_1 q) \end{bmatrix}\right) \\ \left(u\colon \begin{bmatrix} x\colon \mathbf{entity} \\ (\pi_1\pi_2 t)(x) \end{bmatrix}\right) \to \left(v\colon \begin{bmatrix} m\colon M(\pi_1 t) \\ u =_{\begin{bmatrix} x\colon \mathbf{entity} \\ (\pi_1\pi_2 t)(x) \end{bmatrix}} (\pi_2\pi_2 t)(m) \end{bmatrix}\right) \to \mathbf{SendToL\&P}(\pi_1 u, @ : \mathbf{entity}) \end{bmatrix}$$

type checking

$$\dfrac{}{c : \textsc{three}(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q{:} \begin{bmatrix} p{:}\,\textbf{entity} \\ \textbf{Paper}(p)\end{bmatrix} \\ \textbf{Write}(x, \pi_1 q)\end{bmatrix})}\ 2$$

$$\dfrac{\begin{matrix}\vdots \\ \begin{bmatrix} k{:}\,N \\ \begin{bmatrix} A{:}\,\textbf{entity} \to \textsf{type} \\ M(k) \to \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x)\end{bmatrix}\end{bmatrix}\end{bmatrix} : \textsf{type}\end{matrix} \qquad \begin{matrix}\vdots \\ \begin{bmatrix} k{:}\,N \\ \begin{bmatrix} A{:}\,\textbf{entity} \to \textsf{type} \\ M(k) \to \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x)\end{bmatrix}\end{bmatrix}\end{bmatrix}\ true\end{matrix}}{(@ : \begin{bmatrix} k{:}\,N \\ \begin{bmatrix} A{:}\,\textbf{entity} \to \textsf{type} \\ M(k) \to \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x)\end{bmatrix}\end{bmatrix}\end{bmatrix}) : \begin{bmatrix} k{:}\,N \\ \begin{bmatrix} A{:}\,\textbf{entity} \to \textsf{type} \\ M(k) \to \begin{bmatrix} x{:}\,\textbf{entity} \\ A(x)\end{bmatrix}\end{bmatrix}\end{bmatrix}}\ (@)$$

$$\dfrac{(@ : \ldots) : \ldots}{\pi_1\pi_2 t : \textbf{entity} \to \textsf{type}}\ (\Sigma E)* \qquad \dfrac{}{x : \textbf{entity}}\ 1$$

$$\dfrac{\pi_1\pi_2 t : \textbf{entity} \to \textsf{type} \qquad x : \textbf{entity}}{(\pi_1\pi_2 t)(x) : \textsf{type}}$$

$$\dfrac{\textbf{entity} : \textsf{type} \qquad (\pi_1\pi_2 t)(x) : \textsf{type}}{\begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix} : \textsf{type}}\ (\Sigma F), 1$$

$$\dfrac{\begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix} : \textsf{type} \qquad \ldots}{\left(u{:} \begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}\right) \to \left(v{:} \begin{bmatrix} m{:}\,M(\pi_1 t) \\ u = _{\begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}} (\pi_2\pi_2 t)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}}\ (\Pi F)$$

$$\dfrac{\begin{matrix}\vdots \\ \textsc{three}(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q{:} \begin{bmatrix} p{:}\,\textbf{entity} \\ \textbf{Paper}(p)\end{bmatrix} \\ \textbf{Write}(x, \pi_1 q)\end{bmatrix}) : \textsf{type}\end{matrix} \qquad \left(u{:} \begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}\right) \to \left(v{:} \begin{bmatrix} m{:}\,M(\pi_1 t) \\ u = _{\begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}} (\pi_2\pi_2 t)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}}{\begin{bmatrix} c{:}\,\textsc{three}(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q{:} \begin{bmatrix} p{:}\,\textbf{entity} \\ \textbf{Paper}(p)\end{bmatrix} \\ \textbf{Write}(x, \pi_1 q)\end{bmatrix}) \\ \left(u{:} \begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}\right) \to \left(v{:} \begin{bmatrix} m{:}\,M(\pi_1 t) \\ u = _{\begin{bmatrix} x{:}\,\textbf{entity} \\ (\pi_1\pi_2 t)(x)\end{bmatrix}} (\pi_2\pi_2 t)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity})\end{bmatrix} : \textsf{type}}\ (\Sigma F), 2$$

proof search for the first @

$$
\dfrac{
\dfrac{c : \text{THREE}(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q\colon \begin{bmatrix} p\colon \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(x, \pi_1 q) \end{bmatrix})}{\pi_1 c : N}(\Sigma E)
\quad
\dfrac{\textbf{Student} : \textbf{entity} \to \text{type} \qquad \dfrac{c : \text{THREE}(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q\colon \begin{bmatrix} p\colon \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(x, \pi_1 q) \end{bmatrix})}{\pi_1\pi_2\pi_2 c : M(\pi_1 c) \to \begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix}}(\Sigma E)}{(\textbf{Student}, \pi_1\pi_2\pi_2 c) : \begin{bmatrix} A\colon \textbf{entity} \to \text{type} \\ M(\pi_1 c) \to \begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix} \end{bmatrix}}(\Sigma I)
}{
(\pi_1 c, (\textbf{Student}, \pi_1\pi_2\pi_2 c)) : \begin{bmatrix} k\colon N \\ \begin{bmatrix} A\colon \textbf{entity} \to \text{type} \\ M(k) \to \begin{bmatrix} x\colon \textbf{entity} \\ A(x) \end{bmatrix} \end{bmatrix} \end{bmatrix}
}(\Sigma I)
$$

@-Elimination for the first @

$$\dfrac{}{c : \text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q : \begin{bmatrix} p:\textbf{entity}\\ \textbf{Paper}(p)\end{bmatrix}\\ \textbf{Write}(x,\pi_1 q)\end{bmatrix}\right)}\ 2$$

$$\vdots$$

$$\dfrac{(\pi_1 c, (\textbf{Student}, \pi_1\pi_2\pi_2 c)) : \begin{bmatrix} k : N \\ \begin{bmatrix} A:\textbf{entity}\to\textsf{type}\\ M(k)\to \begin{bmatrix} x:\textbf{entity}\\ A(x)\end{bmatrix}\end{bmatrix}\end{bmatrix}}{\pi_1\pi_2 t : \textbf{entity}\to\textsf{type}}\ (\Sigma E)* \qquad \dfrac{}{x : \textbf{entity}}\ 1$$

$$\dfrac{\textbf{entity} : \textsf{type} \qquad \dfrac{\pi_1\pi_2 t : \textbf{entity}\to\textsf{type} \quad x : \textbf{entity}}{(\pi_1\pi_2 t)(x) : \textsf{type}}}{\begin{bmatrix} x:\textbf{entity}\\ (\pi_1\pi_2 t)(x)\end{bmatrix} : \textsf{type}}\ (\Sigma F), 1$$

$$\dfrac{}{u : \begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}}\ 3 \qquad \dfrac{}{c : \text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q : \begin{bmatrix} p:\textbf{entity}\\ \textbf{Paper}(p)\end{bmatrix}\\ \textbf{Write}(x,\pi_1 q)\end{bmatrix}\right)}\ 2$$

$$\vdots$$

$$\left(v : \begin{bmatrix} m : M(\pi_1 c)\\ u =_{\begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}$$

$$\dfrac{\begin{bmatrix} x:\textbf{entity}\\ (\pi_1\pi_2 t)(x)\end{bmatrix} : \textsf{type} \qquad \left(v : \begin{bmatrix} m : M(\pi_1 c)\\ u =_{\begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}}{\left(u : \begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}\right) \to \left(v : \begin{bmatrix} m : M(\pi_1 c)\\ u =_{\begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}}\ (\Pi F), 3$$

$$\vdots$$

$$\dfrac{\text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q : \begin{bmatrix} p:\textbf{entity}\\ \textbf{Paper}(p)\end{bmatrix}\\ \textbf{Write}(x,\pi_1 q)\end{bmatrix}\right) : \textsf{type} \qquad \left(u : \begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}\right) \to \left(v : \begin{bmatrix} m : M(\pi_1 c)\\ u =_{\begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \textsf{type}}{\begin{bmatrix} c : \text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q : \begin{bmatrix} p:\textbf{entity}\\ \textbf{Paper}(p)\end{bmatrix}\\ \textbf{Write}(x,\pi_1 q)\end{bmatrix}\right)\\ \left(u : \begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}\right) \to \left(v : \begin{bmatrix} m : M(\pi_1 c)\\ u =_{\begin{bmatrix} x:\textbf{entity}\\ \textbf{Student}(x)\end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m)\end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity})\end{bmatrix} : \textsf{type}}\ (\Sigma F), 2$$

type cheking

$$\frac{}{v : \begin{bmatrix} m\colon M(\pi_1 c) \\ u = \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} (\pi_1\pi_2\pi_2 c)(m) \end{bmatrix}}\ 4 \qquad c : \text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q\colon \begin{bmatrix} p\colon \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(x, \pi_1 q) \end{bmatrix}\right) \quad u : \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}$$

$$\vdots$$

$$\frac{\begin{matrix}\vdots \\ \begin{bmatrix} m\colon M(\pi_1 c) \\ u = \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} (\pi_1\pi_2\pi_2 c)(m) \end{bmatrix} : \text{type}\end{matrix} \qquad \dfrac{\begin{matrix}\vdots \\ \lambda x.\textbf{Send}(\pi_1 u, x) : \textbf{entity} \to \text{type}\end{matrix} \qquad \dfrac{\textbf{entity} : \text{type} \qquad \begin{matrix}\vdots \\ \textbf{entity}\ \mathit{true}\end{matrix}}{(@ : \textbf{entity}) : \textbf{entity}}\,(@)}{\textbf{Send}(\pi_1 u, @ : \textbf{entity}) : \text{type}}\,(\Pi E)}{\left(v\colon \begin{bmatrix} m\colon M(\pi_1 c) \\ u = \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} (\pi_1\pi_2\pi_2 c)(m) \end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, @ : \textbf{entity}) : \text{type}}\,(\Pi F), 4$$

proof search for the second @

$$\dfrac{c : \text{THREE}\left(\lambda x.\textbf{Student}(x),\ \lambda x.\begin{bmatrix} q : \begin{bmatrix} p : \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(x, \pi_1 q) \end{bmatrix}\right)}{\pi_2\pi_2\pi_2\pi_2\pi_2 c : \begin{bmatrix} z : \begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} \\ \begin{bmatrix} y : M(\pi_1 c) \\ z =_{\begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}} (\pi_1\pi_2\pi_2 c)(y) \end{bmatrix} \end{bmatrix} \to \begin{bmatrix} q : \begin{bmatrix} p : \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(\pi_1 u, \pi_1 q) \end{bmatrix}}\ (\Sigma E)$$

$$\dfrac{\pi_2\pi_2\pi_2\pi_2\pi_2 c : \cdots \qquad u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}}{(\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u) : \begin{bmatrix} y : M(\pi_1 c) \\ z =_{\begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}} (\pi_1\pi_2\pi_2 c)(y) \end{bmatrix} \to \begin{bmatrix} q : \begin{bmatrix} p : \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(\pi_1 u, \pi_1 q) \end{bmatrix}}\ (\Pi E)$$

$$\dfrac{(\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u) : \cdots \qquad v : \begin{bmatrix} m : M(\pi_1 c) \\ u =_{\begin{bmatrix} x : \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m) \end{bmatrix}}{((\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u))(v) : \begin{bmatrix} q : \begin{bmatrix} p : \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(\pi_1 u, \pi_1 q) \end{bmatrix}}\ (\Pi E)$$

$$\dfrac{((\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u))(v) : \cdots}{\pi_1\pi_1(((\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u))(v)) : \textbf{entity}}\ (\Sigma E)$$

Final representation

Three students wrote a paper. They sent it to L&P.

$$\begin{bmatrix} c\colon \begin{bmatrix} k\colon N \\ \begin{bmatrix} k =_N 3 \\ \begin{bmatrix} f\colon M(k) \to \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix} \\ \begin{bmatrix} \textbf{Injection}_{\textbf{E}}(f) \\ \left(z\colon \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}\right) \to \\ \begin{bmatrix} y\colon M(k) \\ z =_{\begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}} fy \end{bmatrix} \to \begin{bmatrix} q\colon \begin{bmatrix} p\colon \textbf{entity} \\ \textbf{Paper}(p) \end{bmatrix} \\ \textbf{Write}(\pi_1 z, \pi_1 q) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \left(u\colon \begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}\right) \to \left(v\colon \begin{bmatrix} m\colon M(\pi_1 c) \\ u =_{\begin{bmatrix} x\colon \textbf{entity} \\ \textbf{Student}(x) \end{bmatrix}} (\pi_1\pi_2\pi_2 c)(m) \end{bmatrix}\right) \to \textbf{SendToL\&P}(\pi_1 u, \pi_1\pi_1(((\pi_2\pi_2\pi_2\pi_2\pi_2 c)(u))(v))) \end{bmatrix}$$

Conclusion

- Constructive GQs by Sundholm (1989) and idea to avoid the proportion problem
- Weak and strong reading of donkey sentences
- Donkey anaphora
- Discourse anaphora and distributive reading

- Allow many proofs for a proposition
- Common nouns are predicates

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